A Fast Optimization Algorithm for Multi-Dimensional RF-Pulse Design under multiple Constraints

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INTRODUCTION: Multi dimensional spatially selective excitation (mDSSE) RF pulse design aims to homogenize the magnetization over a given region of interest. After discretizing the solution of the Bloch Equation under the small flip angle approximation, the problem is to find an optimal numerical solution to the least squares problem argmin$\{\|\mathbf{Ax}-\mathbf{b}\|^2\}$ (**1**) where the matrix **A** is the discretization of the integral operator as in [1], **x** and **b** the vectors corresponding to the requested RF pulses and desired magnetization respectively. In this paper we present an algorithm to solve (**1**) in a fast and stable way. From recent works it appears that a smooth RF pulse profile improves accuracy in the magnetization obtained from a transmit system. The speed achieved by the algorithm is exploited to add regularization terms to (**1**) in order to optimize the smoothness of the solution.

METHODS AND MATERIALS: Regularized RF pulse design: we wish to find the RF pulses for the desired magnetization on a 2D domain corresponding to the central slice of a spherical phantom as plotted in figure 1. We employ a 2-channels birdcage head coil Tx system on a 7T MR scanner (Achieva, Philips Medical Systems). The B_1^+ map are obtained according to [2] (see figure 2 for a picture of the channel 1 map). To solve (**1**) a regularization term must be added and the normal equations become $(\mathbf{A}^H\mathbf{A}+\lambda\mathbf{I})\mathbf{x}_\lambda=\mathbf{A}^H\mathbf{b}$ (**2**). The optimal choice of λ is not straightforward. The approach we follow is to choose a set of possible values Λ and compute the solutions of (**2**) for each $\lambda\in\Lambda$ with a fast and stable multi-shift Conjugate Gradient method (mCGLS) derived in [3]. mCGLS has the great advantage to compute all solutions $\mathbf{x}_\lambda$ *simultaneously*: the amount of computation is comparable to that of a standard CG algorithm for the normal equations (where only one solution is computed). Once all solutions are known, the optimal one can be easily found by plotting the L-curve or a graph of the errors obtained by the corresponding simulated magnetization profiles using a Bloch simulator. We apply mCGLS to the solution of mDSSE RF pulse design problem. The number of rows of matrix **A** and **b** is reduced in such a way that only the voxels inside the ROI are taken into account. In this case the full 2D domain and ROI contains 32^2=1024 and 512 voxels respectively, with a reduction factor of 2 in the rows of **A**. We choose a spiral-in *k*-space trajectory constrained by maximum Gradient amplitude and Slew rate. This result in a pulse length of 3.7 ms. The dimensions of **A** are 512x1132.

Smooth RF pulse design. We apply an extra weight to the least square problem (**1**) and we solve argmin$\{\|\mathbf{A}\,\mathbf{x}_{\lambda,\eta}-\mathbf{b}\|^2+\lambda\,\|\mathbf{x}_{\lambda,\eta}\|^2+\eta\|\mathbf{D}\,\mathbf{x}_{\lambda,\eta}\|^2\}$ (**3**) where **D** is the finite differences second derivative operator. Note that the solution now depends on two parameters. We look for the solution corresponding to the largest allowed value of η such that the obtained magnetization is accurate. Again, we choose a set of values for λ and $\eta\in\Xi$ and compute the solution by mCGLS (mCGLS is now run for each value of η). The computations are carried out with MATLAB 7.4.0 on a Intel Core 2 Duo processor T3400 2.16 GHz.

RESULTS: We set $\Lambda=\{10^{-4}, 10^{-3.75}, 10^{-3.50},\ldots 10^{1.75}, 10^{2}\}$ (25 values) and solve (**2**) running mCGLS. The L-curve is plotted in figure 3: each circle corresponds to a numerical solution for a given value of $\lambda\in\Lambda$. The computation time is 0.54 seconds. The obtained magnetization from a Bloch equation simulator is displayed in Figure 4.

We set $\Xi=\{10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}\}$ and solve (**3**) by mCGLS. Once the solutions are known, a 3D plot of the error obtained by the simulated profiles over the 2D domain Λ x Ξ is displayed (figure 5). The solution fulfilling the accuracy criterion (in this case: $\log_{10}$ (error)≤ 0.90) and maximizing the value of η and λ is indicated on the graph. A comparison between two RF profiles (smoothened and not smoothened) is plotted in figure 6. The total time needed (construction of the constrained *k*-space trajectory, construction and reduction of **A**, running of mCGLS and plotting of the errors' graph) is less than one minute.

Experiment: We compute an optimal RF pulse for an X-shaped magnetization pattern in the center slice of the spherical phantom. One transmit channel is employed. We speed up the *k*-space scan setting $k(t)=k_{\max}\tau(t)^\alpha\exp(i\omega\tau(t))$ and α=3 with ω=2*n*π and *n*=$k_{\max}$ FOV. In this way the *k*-space is undersampled in the higher spatial frequencies and the pulse length is shortened to 8 ms. The dimensions of **A** are 512x1247. The whole computation procedure takes again less than one minute. The pulse is played out on channel 1 of the head coil of the 7T MRI scanner. The resulting magnetization is shown in figure 7.

CONCLUSIONS: We described a method to find quickly an optimal RF pulse from the solution of the regularized least squares problem (**1**). The multi-shift algorithm mCGLS enables us to compute the numerical solution over a large range of regularization parameters in a quick and stable way. The method has been applied for two parameters regularization (**2**) with a computation time of less than one minute for the whole RF pulse design process. The speed up obtained by the application of mCGLS is being exploited by the authors for RF pulse design with reduction of local 1-gram SAR.

References: [1] Grissom W., Yip C., Zhang Z., Stenger V.A., Fessler J. A. and Noll D. C. *Spatial Domain Method for the design of RF Pulses in Multicoil Parallel Excitation* Magn. Reson. Med. 56, 620-629 (2006); [2] Vasily L. Yarnykh Actual Flip-Angle Imaging in the Pulsed Steady State: *A Method for Rapid Three-Dimensional Mapping of the Transmitted Radiofrequency Field* Magn. Reson. Med. 57, 192-200 (2007); [3] Jasper van den Eshof and Gerard L.G. Sleijpen *Accurate gradient methods for families of shifted systems* Applied Numerical Mathematics, 49,17-37 (2004)

Fig 1

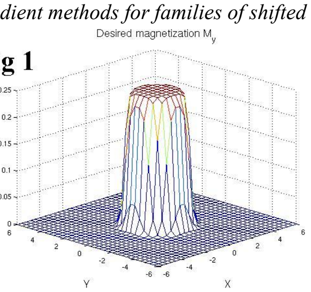

Fig 2

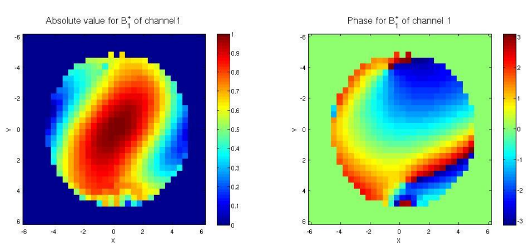

Fig 3

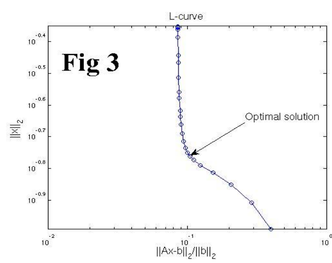

Fig 4

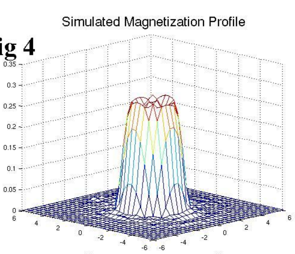

Fig 5

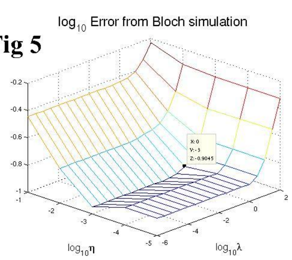

Fig 6

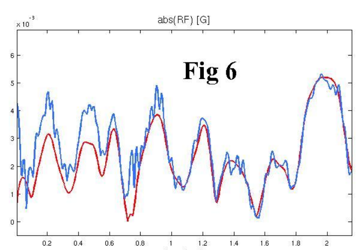

Fig 7