A Fast Algorithm for local-1gram-SAR optimized Parallel-Transmit RF-Pulse Design

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INTRODUCTION: In the design of multi dimensional spatially selective RF pulses particular attention must be paid to the local 1 gram Specific Absorption Rate (1g-SAR). Employing multi-transmit RF systems allows an additional degree of freedom which can be exploited to design RF pulses with lowest maximal 1g-SAR over the whole spatial domain. While the problem of total (global) SAR minimization can be solved quite easily[1], here we present a new method which solves the problem of local SAR optimization in a limited amount of time. For this purpose innovative mathematical techniques are applied to this problem.

METHODS AND MATERIALS: After the discretizing the solution of the Bloch Equation under the small flip angle approximation, the problem is to find an optimal numerical solution to the least squares problem argmin$\{\|\mathbf{Ax}-\mathbf{b}\|^2\}$ (**1**) where the matrix **A** is the discretization of the integral operator as in [2], **x** and **b** are vectors corresponding to the requested RF pulses and desired magnetization, respectively. Due to the typically high condition number of **A,** a regularization term must be added to obtain a reliable solution of (**1**) and the problem becomes argmin$\{\|\mathbf{Ax}-\mathbf{b}\|^2+\lambda\|\mathbf{x}\|^2\}$ (**2**). The weight on $\|\mathbf{x}\|^2$ has a beneficial effect also for SAR optimized solutions, since the *total* SAR is proportional to the squared solution norm (see [3]). However, the *local* 1 gram SAR is of importance, since the following constraint must be fulfilled: $\max_{\mathbf{r}}$1g-SAR(**r**) $\leq SAR_{max}$ with $\mathbf{r} \in$ ROI (the 3D spatial domain). We aim to lower the $\max_{\mathbf{r}}$1g-SAR(**r**) for **x** (denoted by max-1g-SAR(**x**)) while maintaining a good accuracy of the Bloch verified magnetization profile (denoted by `bloch`(**x**)). Analogously to [4] we construct local SAR operators $\mathbf{S_r}$ such that $\|\mathbf{S_r x}\|^2=w(\mathbf{r})\|\mathbf{F_r Z x}\|^2=$SAR(**r**). $\mathbf{F_r}$ are sparse, block-diagonal matrices describing the E-fields, **Z** a permutation matrix and hence $\mathbf{S_r}$ are sparse, block-diagonal matrices and $w(\mathbf{r})=\sigma(\mathbf{r})(2\rho(\mathbf{r}))^{-1}$. To construct a 1g-SAR operator we must average SAR(**r**) over a 1 gram cube around **r**. This is done by computing $\equiv$ `chol`$[\sum_j(w(r)\mathbf{F_r}^H\mathbf{F_r})\,\mathbf{Z}]$ with $j\in J$ an index set such that $\sum_j\sigma(\mathbf{r}_j)=1$g and `chol` the function returning the cholesky factor of a matrix. Then we have: 1g-SAR(**r**)=$\|\ \mathbf{x}\|^2$. We are interested in the solutions of (**2**) which minimize $\max_{\mathbf{r}}\|\ \mathbf{x}\|^2$. The approach we follow is to compute first an optimal solution $\mathbf{x}^{(2)}$ to (**2**). To solve (**2**) we apply the multi-shift mCGLS algorithm derived in [5]. After choosing a set of values $\lambda\in\Lambda$, mCGLS computes each solution $\mathbf{x}_\lambda$ *simultaneously*. The optimal solution can then be found by plotting the L-curve. This solution has a lowest norm and thus optimal w.r.t. total SAR. We then compute the highest local values of 1g-SAR(**r**) obtained by $\mathbf{x}^{(2)}$. The strategy is to lower those high values of 1g-SAR(**r**) and to better distribute the 1g-SAR. We add an extra weighting term to (**2**) and we obtain the following modified problem: $\arg\min_{\mathbf{x}}\{\|\mathbf{Ax}-\mathbf{b}\|^2+\lambda\|\mathbf{x}\|^2+\eta(\alpha_1\|\mathsf{S}_{\mathbf{r}_1}\mathbf{x}\|^2+\alpha_2\|\mathsf{S}_{\mathbf{r}_2}\mathbf{x}\|^2+\ldots+\alpha_R\|\mathsf{S}_{\mathbf{r}_R}\mathbf{x}\|^2)\}$ (**3**) where $\mathbf{r}_j$ denotes a voxel from the *R* top values of 1g-SAR($\mathbf{x}^{(2)}$). To solve (**3**) we apply again the multi-shift mCGLS and adapt it to the case of two regularization parameters, $\lambda\in\Gamma$ and $\eta\in\Xi$. During the iterative algorithm, the weights $\alpha_j\equiv\|\ _j\mathbf{x}\|^2$ are computed every *n* steps. In this way, we allow more freedom to the lower 1g-SAR values and less to the higher values to homogenize the 1g-SAR of the *R* reference voxels. The two matrix-vector multiplications employing the matrix $[\mathbf{A}^T\ \ _1^T\ \ldots\ \ _R^T\ ]^T$ required for each iteration step in mCGLS, can be carried out in an efficient way due to the sparse structure of the matrices: this fact, together with the speed up achieved by mCGLS determine a fast computation of the whole procedure. The computations and simulations are carried out with MATLAB 7.4.0 on a Intel Core 2 Duo processor T3400 2.16 GHz. **RESULTS:** We want to find the RF pulses for the desired magnetization (flip angle 15º) on a 2D domain corresponding to the central slice of human head (see figures 1 and 2). We employ the E-field maps, B_1^+ maps and conductivity maps over the whole 3D spatial domain (715k voxels) obtained by FDTD simulations for a 12-channels 7 T (300MHz) head coil loaded with the Hugo Model. We choose a spiral-in *k*-space trajectory constrained by the maximum Gradient amplitude and Slew rate and 3-fold radial undersampling. This results in a pulse length of 2.0 ms. The dimensions of **A** are 641 x 3756. We set $\Lambda=\{10^{-4}, 10^{-3.75}, 10^{-3.50},\ldots 10^{1.75}, 10^{2}\}$ (25 values) and solve (**2**) by running mCGLS. The best solution $\mathbf{x}^{(2)}$ (error from `bloch` ($\mathbf{x}^{(2)}$)=0.130) is found on the L-curve. We compute 1g-SAR($\mathbf{x}^{(2)}$) over the 3D ROI and find max-1g- SAR($\mathbf{x}^{(2)}$)=1.49 [W/kg]. We take the 10 voxels (*R*=10) corresponding to the 10 highest values of 1g-SAR and, after constructing $_j$, j=1...10, we solve (**3**) setting $\Xi=\{10^{-3}, 10^{-2.75}, 10^{-2.50},\ldots 10^{-.75}, 10^{-1}\}$ (9 values) and updating the weights α_j every 10 iteration steps (*n*=10). Note that the solution now depends on two parameters. The obtained solution $\mathbf{x}^{(3)}$ corresponding to the largest allowed value of η was found to be as accurate as $\mathbf{x}^{(2)}$ (error from `bloch` ($\mathbf{x}^{(3)}$)=0.126). The resulting 1g-SAR($\mathbf{x}^{(3)}$) was found to be max-1g-SAR($\mathbf{x}^{(3)}$)=1.15 [W/kg]: a reduction for max-1g-SAR of 23% w.r.t. $\mathbf{x}^{(2)}$. To see how the local SAR is better distributed look at figure 3 where the 100 highest 1g-SAR(**r**) values are plotted for both solutions: the local SAR optimized solution gives rise to a more homogeneous distribution. This fact is evident from an in-slice plot of the 1g-SAR (see figure 4). The two RF waveforms and the simulated magnetization profile are displayed in figures 5 and 6. The numerical solution of (**2**) and (**3**) takes in total 3 minutes. The construction of the $_j$ matrices takes 10 seconds, the computation of 1g-SAR over the 3D whole spatial domain takes about 9 minutes (this step is computed twice, once for $\mathbf{x}^{(2)}$ and once for $\mathbf{x}^{(3)}$. **CONCLUSIONS:** An algorithm to quickly design local 1g-SAR optimized RF pulses was described. The multi-shift algorithm mCGLS applied to a SAR distribution homogenization strategy makes a reduction of the maximum 1g-SAR of about 23% in a relatively short time possible, keeping in mind the performance limitation of MATLAB programming environment. Implementation of the algorithm in the C programming language and parallelization of the computations will speed up the whole process to achieve real time computations. **References**: [1] Lattanzi et al. Magn. Reson. Med. 61:315-334 (2009) [2]Grissom W. et al, *Spatial Domain Method for the design of RF Pulses in Multicoil Parallel Excitation* Magn. Reson. Med. 56, 620-629 (2006) [3]U. Katscher and P. Börnert *Parallel RF transmission in MRI* NMR Biomed. 19: 393-400 (2006);[4] A.C. Zelinski et al. *Designing RF pulses with optimal specific absorption rate characteristics and exploring excitation fidelity, SAR and pulse duration tradeoffs* Proc. Intl. Soc. Mag. Reson. Med. (2007); [5]J. van den Eshof and Gerard L.G. Sleijpen *Accurate gradient methods for families of shifted systems* Applied Numerical Mathematics, 49,17-37 (2004);

Fig 1

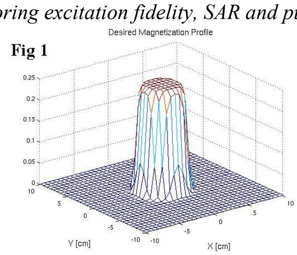


Fig 2

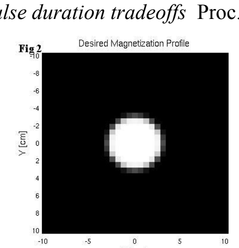
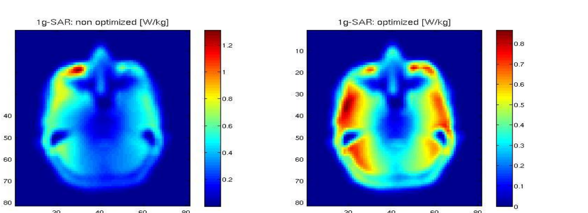


Fig 4



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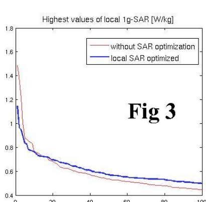
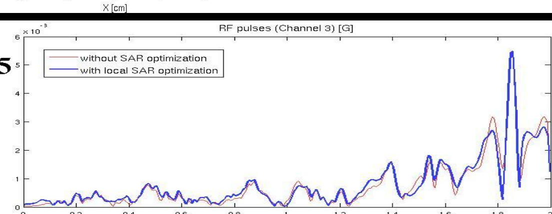


Fig 5



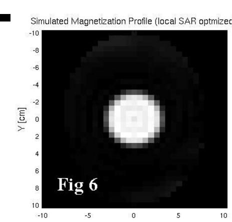


Fig 6