Sparse parallel transmission using optimized sparse k-space trajectory by simulated annealing

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Introduction: The combination of sparse pulses and parallel transmission [1, 2] can further reduce the excitation duration using both coil sensitivity and sparse *k*-space sampling. In this work, a novel sparse parallel transmission design is proposed to further shorten the pulse duration by using an optimal *k*-space trajectory. The *k*-space is firstly sampled using the Monto-Carlo sampling schemes [3]. Then an optimized *k*-trajectory traveling through all these samples is designed using the simulated annealing (SA) algorithm. Finally the gradient waveforms and parallel transmission pulses is designed and the feasibility of this method is verified using Bloch simulation.

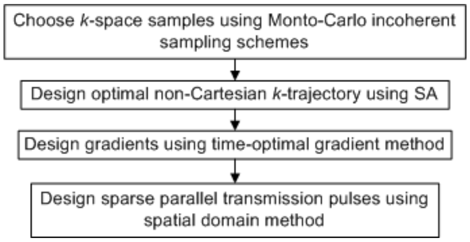

Fig. 1 Sparse transmission pulses design procedure

Theory and method: Fig.1 describes the design procedure of our proposed method. The Mote-Carlo incoherent sampling strategy was firstly used to sample the *k*-space [3]. Then the SA was used to design an optimal single-shot *k*-space trajectory traveling through all these samples. The *k*-space trajectory was initiated by connecting all the samples using a curve and the cost function of the SA was defined as the curve length:

$f=\sum_{j=0}^{N_s-1}\left|k_{j+1}-k_j\right|$ (1), where N_s is the number of the *k*-space samples and the k_j denotes the j_{th} sample on the *k*-space. The optimization is performed by randomly changing the sequence of two *k*-space samples on the trajectory therefore the trajectory and its length was changed. Here, the cost function values before and after changing were denoted by f_1 and f_2 respectively, and the Boltzmann probability distribution was calculated by:

$p=\exp((f_1-f_2)/k_BT)$ (2), where T is the temperature of the system and k_B is the Boltzmann's constant that relates the temperature to the cost function. Whether to accept the new trajectory was determined by comparing a random number r and the Boltzmann probability p: if $r<p$ the new trajectory would be accepted otherwise rejected. It is noticed that if the length of the new trajectory f_2 was shorter than the previous one f_1, the new trajectory would always be accepted due to p was always greater than 1. If f_2 was larger than f_1, there was still some possibility for f_2 to be accepted. By utilizing this general annealing scheme, which usually took a downhill step while sometimes took an uphill step, the SA had the ability to break away from local minimum and to search the whole space for a global optimal solution. The flowchart of optimizing *k*-trajectory using the SA is shown in Fig.2.

After designing this optimal sparse *k*-space trajectory, the corresponding gradient waveforms were designed using the time-optimal gradient method [4] which is able to design gradients with minimum time for arbitrary trajectories. Then the parallel transmission pulses are designed using the spatial domain method [5].

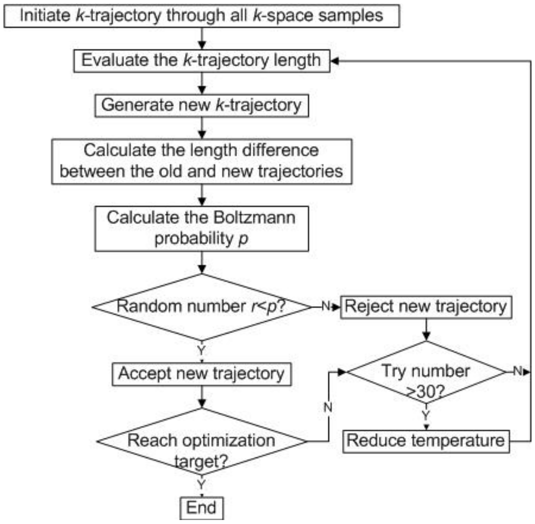

Fig. 2 Flowchart of optimal *k*-trajectory using SA

Simulation and results: An example of the sparse parallel transmission pulses design was simulated using Matlab. The desired excitation pattern was a cylinder with 9 cm diameter and the flip angle is 90°. An 8-element cylindrical coil array was used for excitation and the sensitivity pattern of each element was assumed to be the reciprocal of the distance to the center of the FOV (Fig.3). The reduction factor was 4. The *k*-space extension was 0.5 cycle/cm and 266 samples were chosen using the random undersampling schemes. The optimal *k*-space trajectory, its corresponding gradient waveforms and RF pulses are shown in Fig. 4. Both the gradients and the RF pulses satisfied the limitation of hardware. It is noticed that the rapid change of the gradient waveforms present a challenging requirement of the hardware equipment. The simulation results of the excitation profiles of 8 individual RF pulses and the combined excitation profile are shown in Fig. 5.

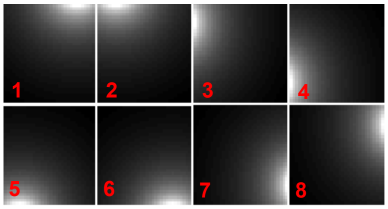

Fig. 3 Sensitivity patterns of 8 elements.

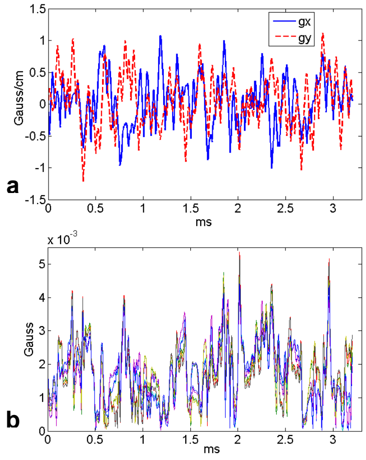

Fig. 4 (a) *x* (blue) and *y* (red) gradients; (b) 8 individual RF pulses ini different colors.

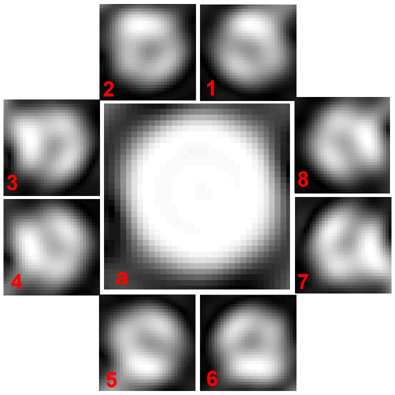

Fig. 5 Excitation profiles of 8 individual RF pulses and (a) their combined profile. The numbers denote channel number.

Conclusion and discussion: The method of designing sparse parallel transmission RF pulses on an optimized sparse *k*-space has been proposed. The optimal *k*-trajectory traveling through the sparse *k*-space samples is designed using the SA algorithm, shortening the corresponding gradient waveforms and pulse width. Bloch simulation has demonstrated the feasibility of this method. The small ripples on both the in-slice and out-of-slice regions are due to the imperfection of the *k*-space undersampling and RF pulses, both need to be improved for better excitation accuracy. The rapid changing gradients require high performance of the gradient coils. Furthermore, this method can also be easily applied to non-parallel pulse design and 3D spatial selective pulse design.

References: [1] Zelinski AC, et al, IEEE Trans Med Imaging 2008; 27: 1213-1229. [2] Chen D, et al, ISMRM 2009: p171. [3] Lustig M, et al, Magn Reson Med 2007; 58: 1182-1195. [4] Lustig M, et al, IEEE Trans Med Imag 2008; 27: 866-873. [5] Grissom W, et al, Magn Reson Med 2006; 56: 620-629.

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