A simple and analytical way to correct for ΔB0 inhomogeneity in the evaluation of B1 maps relying on flip angle measurements and non-selective square pulses

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Introduction: Efficient mitigation of the radiofrequency (RF) inhomogeneity at high field using coil arrays relies on the accurate knowledge of the individual B_1 maps. Based on these maps, algorithms using diverse k-space trajectories can return RF waveforms which homogenize the flip angle (FA). The algorithms initially aimed at counteracting the B_1 inhomogeneity only [1, 2], until the spatial domain approach was introduced to include B_0 variations as well [3]. To date, no simple recipe has been formulated to correct for the latter in the evaluation of the B_1 maps themselves. Here we derive a simple analytical approximation to increase the accuracy of the B_1 mapping techniques which rely on the measurement of the FA using non-selective square pulses, in the presence of B_0 inhomogeneity.

Theory: For a spin off-resonance due to a field offset ΔB_0, the dynamics of a spin under the action of a square RF pulse of field amplitude B_1 and of duration T can be solved analytically. If the magnetization vector initially starts along the z-axis, neglecting relaxation during the RF pulse, the FA is given by the following exact expression:

$$FA = \arccos(1-(\alpha^2/2\Delta^2)\sin^2\Delta) \quad (1)$$

Where $\alpha=\gamma|B_1|T$, $\Delta=(\alpha^2+\delta^2)^{1/2}/2$ and $\delta=\gamma\Delta B_0 T$. All angles are in radians and γ is the gyromagnetic ratio. The angle α would be equal to the measured FA if the pulse was at resonance, i.e. if $\delta=0$. The problem we address here is how to extract in a simple way B_1 from a measurement of the FA and of ΔB_0? From this knowledge, it is of course possible to fit the B_1 value using Eq. (1). A loop over each voxel would need to be performed. But this could be done in a simpler way. Assuming α and δ are not too large, we first perform a series expansion of Eq. (1) with respect to α up to third order using Mathematica (Wolfram Research, Champaign, USA):

$$FA \approx -a\alpha^3 + b\alpha \quad (2)$$

Where $a=\delta^2/240$ and $b=((2-2\cos(\delta))/\delta^2)^{1/2}$. Knowing FA and δ via previous measurements, α and thus $|B_1|$ can be recovered by solving this cubic equation whose roots are given by Cardano's formula [4]. For most practical cases, it can be shown that there are three real and unequal roots and therefore identifying the right solution for each voxel would be time consuming. This can be solved by a perturbation approach by noting that, again for practical cases, the linear term dominates the cubic one. In that case, we obtain as an approximate solution:

$$\alpha \approx FA/b + aFA^3/(b^4-3abFA^2) \quad (3)$$

From which $|B_1|$ can be deduced. As expected, because $b\leq1$, one can see from the leading (first) term of the above equation that one underestimates the B_1 amplitude if ΔB_0 is not taken into account. The accuracy of this correction will depend on the values of $|B_1|$ and $|\Delta B_0|$. For $\alpha = \delta = \pi/2$, Eq. (1) in fact predicts a rough 13 % underestimation of the RF field amplitude if it is merely assumed that α = FA. Such a value for δ is very plausible since pulse lengths can reach up to 1 ms to obtain a reasonably large FA throughout the volume while different susceptibilities in the brain can yield resonance offsets of around 250 Hz at 7T [5]. Using only the first term of Eq. (3) yields a 3 % error while the same full equation returns a 0.2 % error in the estimation of the $|B_1|$ value, hence almost a perfect correction. Moreover, the computation of α consists of elementary operations on the measured FA and ΔB_0 maps and therefore does not require any sophisticated fit over all the voxels.

The more accurate computation of $|B_1|$ can then be used to better evaluate the phase of the RF field too. At the end of the pulse, the phase of the spin φ_{spin} is related to the phase of the RF field φ_{RF} via the following formula: $\varphi_{RF} = \varphi_{spin} - \arctan2(-2\Delta\cos\Delta, \delta\sin\Delta)$, where the function arctan2(y,x) calculates arctan(y/x) and returns an angle in the correct quadrant. For small δ and α this simply yields: $\varphi_{RF} \approx \varphi_{spin} + \pi/2 - \delta/2$, which is a well known result obtained in the small tip angle approximation. Using this approximation however would still yield a 10° phase error for $\alpha = \delta = \pi/2$ so that the use of the exact formula above is recommended.

Measurements and Simulations: We used the modified AFI sequence [6, 7] to measure B_1 and ΔB_0 maps on an 16 cm diameter water phantom using a Siemens Magnetom 7T scanner (Erlangen, Germany) equipped with an 8 channels parallel transmission system. A 900 µs square pulse at full power was used for each channel measurement. The resolution of the acquired maps was $6 \times 6 \times 6$ mm^3 and matrix size $32 \times 32 \times 32$. Based on this experimental data, we simulated the Bloch equation in 2 scenarios. In each case, we used a 2-D spiral k-space trajectory (no acceleration factor, pulse duration = 7.08 ms) and designed an RF pulse whose target excitation pattern is given in the central slice of the phantom in Fig. 1.b, i.e. 18° flip angle in the lower right quadrant (where the B_0 offset is the largest, see Fig. 1.a.) and zero elsewhere. For that purpose, we used the spatial domain method with a Tikhonov regularization and included B_0 inhomogeneity [3]. The two simulations differed only in the B_1 maps that were used to do the RF pulse design. In the first case, the B_1 maps were extracted from the measured FA maps by assuming resonance while in the second case, we used the theory described above to better assess B_1. Once the pulses were computed, a Bloch simulator was used to calculate the resulting FA maps using the true corrected B_1 maps, to study the impact of not using them in the pulse design. The results are illustrated in Fig. 1.c and d. In these figures are shown the absolute differences with the target FA. When ΔB_0 is not taken into account in the estimation of the B_1 maps, absolute errors can reach up to 2°, i.e. more than 10 % error with respect to the target FA, while the deviation does not exceed 3 % (Fig. 1.d.) when the initial B_1 maps are correctly estimated.

Conclusion: We have derived an analytical formula allowing a ΔB_0 correction in the evaluation of the B_1 maps based on the measurement of the FA using non-selective square pulses, decreasing an initial 13 % error in the field amplitude to a 0.2 % error in some possibly encountered cases. Due to its simplicity, such correction is easy to implement and should increase the accuracy of the B_1 maps and thus of the available control at high field. It should in fact prove to be essential if fatty tissues are contained in the volume where the FA is designed to be uniform.

References: [1] U. Katscher et al., MRM 2003; 49:144-150, [2] Y. Zhu, MRM 2004; 51:775-784, [3] W. Grissom et al., MRM 2006; 56:620-629, [4] W. Dunham, Ch. 6 in *Journey through Genius: The Great Theorems of Mathematics*, pp. 133-154, 1990, [5] N. Boulant et al., MRM 2009; 61: 1165:1172, [6] V. L. Yarnykh, MRM 57: 192-200 (2007), [7] A. Amadon et al., Proc. of the 16th ISMRM meeting (2008), p. 1248.

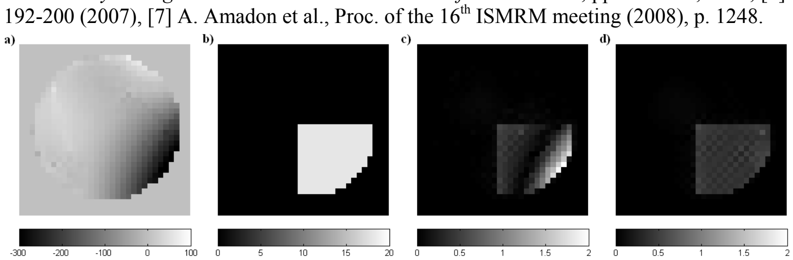

Fig. 1: a) ΔB_0 map measured for the central slice of the phantom with respect to the carrier frequency (in Hz), b) target FA (18° in the lower right quadrant and 0° elsewhere), c) simulated absolute value of the FA error obtained when the evaluation of the B_1 maps does not take into account ΔB_0, and d) absolute value of the FA error with the corrected B_1 maps. The maximum absolute errors in c) and d) are around 2° and 0.5° respectively.