Generalized PRUNO Kernel Selection by Using Singular Value Decomposition (SVD)

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INTRODUCTION Parallel Reconstruction Using Null Operations (PRUNO) is an iterative k-space based reconstruction method for Cartesian parallel imaging. One particular challenge in PRUNO is to select a set of proper nulling kernels. A group of poor kernels will lead to an ill-posed system and affect the accuracy and convergence of the algorithm. The early reported kernel selection method is similar to GRAPPA, in which kernel templates are manually assigned by assuming unbiased symmetry among all coil sensitivity maps [1, 3]. In this work, we demonstrate an improved kernel selection strategy to create generalized PRUNO kernels from the Singular Value Decomposition (SVD) of calibration data. Furthermore, by introducing composite kernels prior to the conjugate-gradient (CG) reconstruction, the reconstruction time wouldn't increase much when a large number of kernels are used. These new strategies boost the robustness of PRUNO with faster algorithm convergence and lower noise sensitivity.

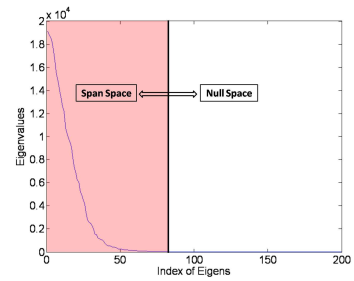

Figure 1: Eigenvalues of calibration vector space.

METHODS In parallel imaging, k-space data among all receiver channels yield strong shift-invariant local linear dependence due to the nature of smoothness of coil sensitivities. Mathematically speaking, the linear span of windowed k-space samples, or local subsets, will be rank deficient if the subset window reaches a certain size. In another word, there exist small convolution kernels, each of which can be used to null all k-space locations through filtering and cross-coil summation. This property is utilized in PRUNO to perform a k-space reconstruction [1]. After finding kernel coefficients through data calibration, a system equation can be formulated as $\mathbf{N}\mathbf{d} = \mathbf{0}$ (1), where each row of **N** corresponds to one PRUNO kernel. And **d** is a vector concatenated from all k-space samples. With further decomposition, the k-space reconstruction can be accomplished by solving all missing samples $\mathbf{d}_\mathbf{m}$ from $(\mathbf{N}_\mathbf{m}^\mathbf{H}\mathbf{N}_\mathbf{m})\mathbf{d}_\mathbf{m} = -\mathbf{N}_\mathbf{m}^\mathbf{H}\mathbf{N}_\mathbf{a}\mathbf{d}_\mathbf{a}$ (2). Here the superscript **H** refers to Hermitian transpose, and the subscripts **m** and **a** represent *missing* and *acquired*, respectively. Multiple kernels are required to make Eq. [2] overdetermined in dimension. These kernels need to preserve good orthogonality and unbiasedness as well to ensure the system well-conditioned. Template based kernel selection is limited if a large number of kernels are desired for improving the system condition. An optimal kernel selection strategy is to acquire generalized kernels directly from the null space of subset samples. This can be feasibly calculated by performing an SVD on the calibration data. One example is shown in Fig 1. In this case, a full kernel width of 5 was used for 8-channel k-space data. Around 1000 k-space subsets (vectors) were collected from the calibration region with a length of 200 for each. SVD was then applied onto these vectors. The eigenvalue plot in Fig 1 implies that the span of the vector space is in very low rank and we can approximate its null space by thresholding small eigenvalues.

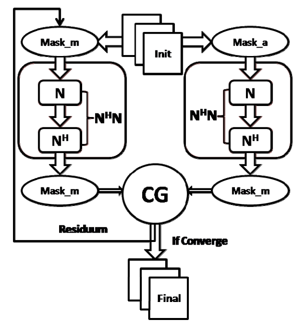

Figure 2: The iterative algorithm for PRUNO reconstruction. Here **Mask_m** and **Mask_a** are masking matrices. It can be sped up if $\mathbf{N}^\mathbf{H}\mathbf{N}$ is precomputed.

The PRUNO reconstruction algorithm is shown in Fig 2. Because each matrix multiplication in Eq. 2 is equivalent to a k-space convolution followed by summation, this system can be solved iteratively without performing any matrix inversion, which is similar to non-Cartesian SENSE reconstruction [4]. However, as the number of PRUNO kernels increase, the number of convolution operations will grow proportionally as well, which will significantly increase the computational time. One alternative solution is to compute composite kernels ($\mathbf{N}^\mathbf{H}\mathbf{N}$) prior to the CG algorithm. Although the computational load would be slightly increased due the increased size of composite kernels, the reconstruction time is almost independent with the number of PRUNO kernels.

This improved PRUNO method is compared with GRAPPA and original PRUNO on both simulated phantom and *in vivo* parallel imaging data. A matrix size of 256 and 8 channels were used in both experiments. The *in vivo* data was acquired from a GE 1.5T Scanner by using a GRE sequence. An accelerating factor of 4 was used to undersample the k-space data along the phase encoding direction. 3 ACS lines were added for the phantom data and 6 ACS lines were added for the *in vivo* data. For both experiments, a kernel width of 5 was used for all PRUNO reconstruction. 8 GRAPPA-like kernels were used to implement the original PRUNO. For the improved PRUNO, 120 generalized SVD kernels were used on the phantom data and 80 were used on the *in vivo* data. To compare the speed of convergence, only 40 PRUNO iterations were executed on phantom data.

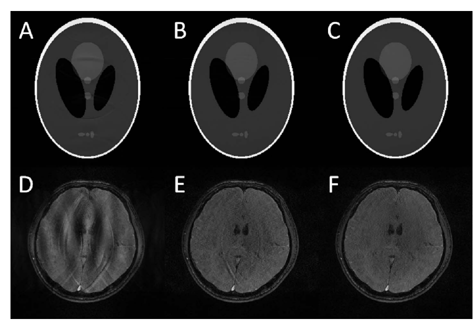

Figure 3: Reconstructed images at 4-fold.
A and D: GRAPPA reconstruction;
B and E: PRUNO with template based kernels;
C and F: PRUNO with SVD based kernel.

RESULTS Fig 3 compares the three reconstruction methods. It shows clearly that PRUNO produces much better images than GRAPPA with the same ACS. With more generalized nulling kernels, the improved PRUNO tends to converge faster than the old one (Fig 3B & 3C). And these new kernels help enhance the robustness of the algorithm against noises (Fig 3E & 3F).

DISCUSSION AND CONCLUSION We have demonstrated a generalized kernel selection strategy to improve the robustness of PRUNO reconstruction. An improved reconstruction process is also presented to reduce the reconstruction time. These new strategies make PRUNO more robust in both algorithm speed and image quality. This will make PRUNO an indeed promising technique to perform fast parallel imaging at high accelerating rate.

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