Homotopic l_0 minimization technique applied to dynamic cardiac MR imaging

M. Usman[1], and P. Batchelor[1]

[1]King's College London, London, London, United Kingdom

Introduction: The l_1 minimization technique has been empirically demonstrated to exactly recover an S-sparse signal with about 3S-5S measurements [1]. In order to get exact reconstruction with smaller number of measurements, recently, for static images, Trzasko [2] has proposed homotopic l_0 minimization technique. Instead of minimizing the l_0 norm which achieves best possible theoretical bound (approximately 2S measurements) but is a NP hard problem or l_1 norm which is a convex optimization problem but requires more measurements, the homotopic technique minimizes iteratively the continuous approximations of the l_0 norm. In this work, we have extended the use of homotopic l_0 method to dynamic MR imaging. For dynamic 2D CINE data, using five different non-convex functional approximations to l_0 norm, we have compared the performance of homotopic l_0 minimization technique with the standard l_1 method.

Method: Given a linear system of equations expressed as: **Au= b,** where **u** is sparse signal to be recovered, **A** is a measurement matrix with dimensions MxN (M<<N), **b** is a set of M measurements; the homotopic l_0 minimization problem is solved by iteratively solving a sequence of problems given by

$$u^{t+1} = \underset{u^t}{\operatorname{argmin}} \sum_{x\in\Omega} \rho(|u^t(x)|, \sigma^t) \quad \text{s.t.} \quad Au^t = b$$

where u^t is the recovered sparse signal in the iteration't', Ω is a set of indices on which **u** resides, $\sigma^{t+1} < \sigma^t; \rho(|u^t(x)|, \sigma^t)$ is a functional that approximates the l_0 norm with the following property: $\lim_{\sigma\to 0} \sum_{x\in\Omega} \rho(|u(x)|, \sigma) = \|u(x)\|_0 = \sum_{x\in\Omega} 1(|u(x)| > 0)$, where **1** is the indicator function. Over the iterations, as $t \to \infty, \sigma^t \to 0$. To evaluate the performance of homotopic l_0 technique in dynamic cardiac MRI, we have used following five non-convex functionals [2][3] that approximate the l_0 norm.

1) Laplace function : $\rho(|u(x)|, \sigma) = 1 - e^{-\frac{|u(x)|}{\sigma}}$; 2) Geman function : $\rho(|u(x)|, \sigma) = \dfrac{|u(x)|}{u(x) + \sigma}$

3) Concave logrithmic penalty : $\rho(|u(x)|, \sigma) = \log(1 + \dfrac{|u(x)|}{\sigma})$; 4) Power function penalty : $\rho(|u(x)|, \sigma) = |u(x)|^p, 0 \le p \le 1$

5) Arctan function : $\rho(|u(x)|, \sigma) = \arctan(\dfrac{|u(x)|}{\sigma})$

With Philips 1.5 T scanner, we acquired single slice multiple time frames of 2D CINE data with following parameters: SSFP sequence, FOV: 300X300 mm^2, TE/TR: 1.46/3 ms, voxel size: 0.89 mm x 1.68 mm, acquisition matrix size: 336x178x48. The under-sampling was simulated by randomly skipping the phase encode lines in k-space for each time frame. The x-f space corresponding to each frequency encoding position was independently reconstructed with l_1 minimization [4] and homotopic l_0 method. The reconstruction in every iteration of homotopic l_0 method was done via fixed point solver based on conjugate gradient method [2]. For each functional, the regularization parameters involved in the homotopic method were manually tuned to get optimal performance.

Results: From the reconstructed cardiac frames (Fig.1a) with under-sampling factor of 10, the standard l_1 method exhibits significantly worse spatial resolution than the homotopic method. The temporal resolution however is nearly the same for both l_1 and homotopic methods (Fig.1b). From the error images shown in Fig.1, the error for l_1 minimization method is more concentrated around high contrast regions and edges, while for homotopic method with non-convex functionals, the error is more uniform across the image. For all non-convex functionals used, the homotopic l_0 method gives similar reconstruction performance.

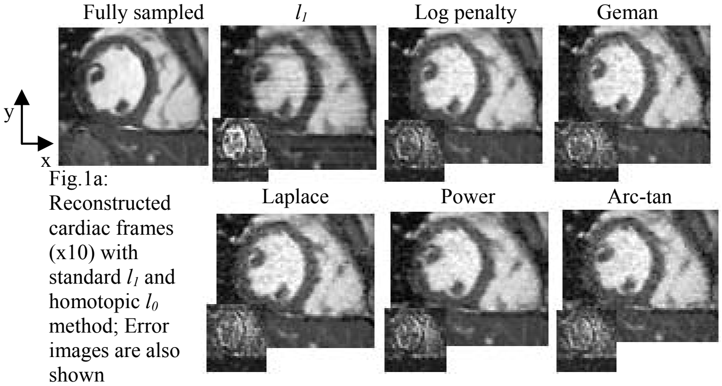

Fig.1a: Reconstructed cardiac frames (x10) with standard l_1 and homotopic l_0 method; Error images are also shown

Discussion: In order to explain, why we get better spatial resolution and nearly the same temporal resolution for reconstructed dynamic cardiac MR data using homotopic l_0 method when compared with standard l_1, consider the reconstruction error images in x-f space corresponding to a dynamic region in FOV [Fig.2]. These error images are shown on same scale and are enhanced to visualize details. At DC frequency (f=0), the reconstruction error for homotopic method is significantly less than for l_1 method. However, at high temporal frequencies, the reconstruction error is nearly the same for both methods. Hence, the homotopic approach primarily brings improvement in static parts of FOV. Over all, this results in better spatial resolution but nearly the same temporal resolution. Future works involve improvement of homotopic method to achieve better temporal resolution.

Conclusion: For dynamic MR CINE data, the homotopic l_0 minimization technique using different non-convex functionals reconstructs sequence of dynamic cardiac frames with better spatial resolution than the standard l_1 method.

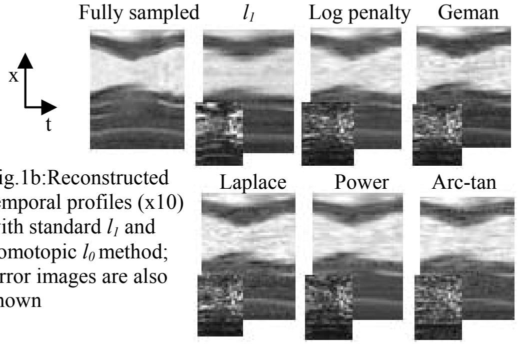

Fig.1b:Reconstructed temporal profiles (x10) with standard l_1 and homotopic l_0 method; Error images are also shown

References:[1] E. Candès et al, Proc. SPIE conf. 5914, 2005 [2] J. Trzasko et al, IEEE Trans. Med. Imaging, vol. 28, pp. 106121, 2009 [3] E. Candès et al, Journal of Fourier analysis and applications,14(5-6):877-905, 2008. [4] Gamper et al. Magn. Reson. Med. 59,365-373, 2008

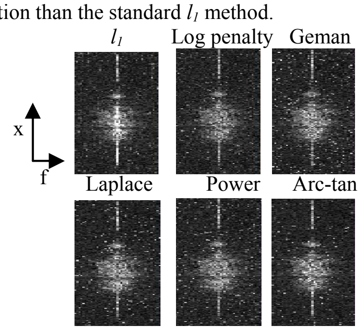

Fig.2: Error images (in x-f space domain) with acceleration factor of 10; the images are shown on same scale and enhanced to visualize details.