Time dependent regularization for functional magnetic resonance inverse imaging

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INTRODUCTION

Magnetic resonance Inverse Imaging (InI) is a recently proposed framework for ultrafast MR acquisition [1,2,3]. A highly parallel radio-frequency coil array is utilized for spatial encoding of the data along some directions, omitting the standard phase encoding. For the InI encoded directions, the image reconstruction leads to a linear inverse problem, the solution of which is not unique and potentially sensitive to the measurement noise. Consequently, the problem necessitates regularization in order to obtain stable solutions. The optimal degree of regularization depends on the signal-to-noise ratio (SNR) of the data, which varies drastically across time for functional acquisitions: at baseline period, the signal consists mainly of noise, and after stimulus presentation the hemodynamic response leads naturally to a temporally varying SNR. In this work, we propose a method for time dependent regularization for functional InI, which is adapted from a recently presented generalization of Kalman filtering for temporally varying observation noise [4].

METHODS

The image reconstruction in InI corresponds to a linear regression model $\mathbf{Y}(t)=\mathbf{A}\mathbf{X}(t)+\mathbf{n}$, $t=1,\ldots,T$. Such a linear model is solved independently for each Fourier encoded spatial location. The forward matrix $\mathbf{A}$ is obtained by performing a reference scan, the $\mathbf{X}(t)$ is a vector of spatial intensities to be estimated, and the noise is assumed to Gaussian with zero mean and covariance $\mathbf{C}$ for all of time-points $t=1,\ldots,T$. Because of the InI acceleration, the matrix $\mathbf{A}$ cannot be directly inverted and the Maximum Likelihood (ML) estimate for $\mathbf{X}(t)$ does not exist in a meaningful sense. In the Bayesian approach the situation is remedied by assuming a zero-mean Gaussian prior for $\mathbf{X}(t)$ with time varying covariance $\gamma(t)\mathbf{1}$. If the regularizing prior variance $\gamma(t)$ is known, the inverse solution can be obtained by classical Minimum-Norm Estimate (MNE): $\mathbf{X}_{MNE}(t) = \gamma(t)\mathbf{A}^T(\gamma(t)\mathbf{A}\,\mathbf{A}^T+\mathbf{C})^{-1}\,\mathbf{Y}(t)$, which corresponds to the Maximum A Posteriori (MAP) estimate under the modeling assumptions. Typically, the parameter $\gamma(t)$ is not known and here we adopt the Variational Bayesian (VB) approach for joint estimation of $\mathbf{X}(t)$ and $\gamma(t)$, which assumes a separable approximation for the joint posterior probability density $p(\mathbf{X}(t),\gamma(t)|\mathbf{Y}(t))\approx Q(\mathbf{X}(t))Q(\gamma(t))$. If we take the asymmetric Kullback-Leibler (KL) divergence as a distance metric for the probability density approximation, it turns out that the $Q(\mathbf{X}(t))$ is a Gaussian and $Q(\gamma(t))$ is an Inverse-Gamma distribution [4]. To obtain a robust estimate for the time varying regularization, we assume a "heuristic" dynamic model for the parameters: at each step, the estimated Inverse-Gamma distribution is used as a prior for the next time point, with increased variance to facilitate the dynamical behavior [4]. We test the method with a simulated dataset where a 32-channel InI setup is used to reconstruct a (64x64) 2D slice from 1D (64X1) gradient encoded signals.

RESULTS

The time courses of the regularization parameters demonstrate the capability of the method in adapting to temporally varying SNR (see, Figure 1). The estimated regularization parameters clearly reflect the overall power of the activation across time. In comparison with standard MNE, the dynamical dMNE suppresses the contributions of noise in the baseline period, facilitating the estimation of the activation onset latency. The method also automatically adapts to the overall magnitude of the simulated hemodynamic response (case #1 vs. case #2).

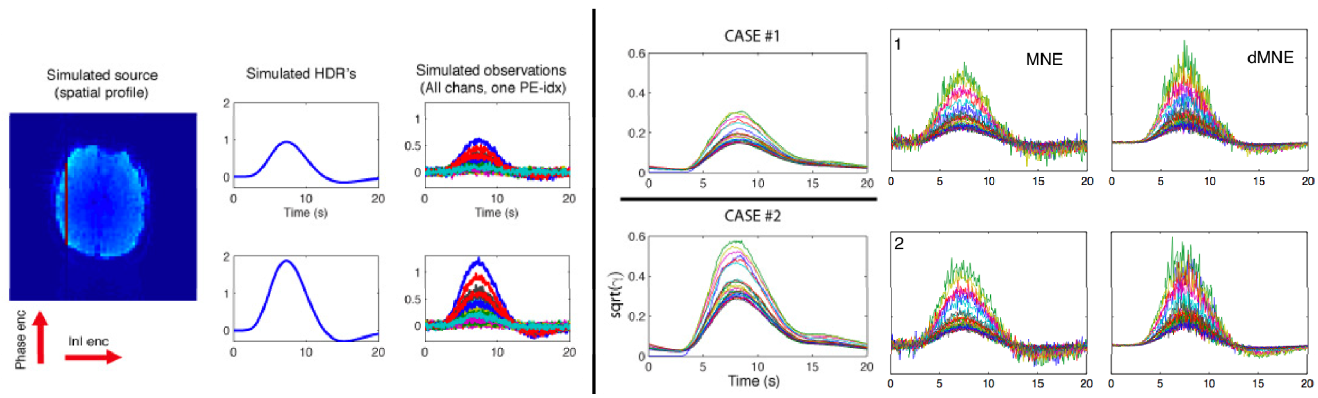

Figure 1 *The left panel shows the simulated activation (red line) on top of the sum-of-squares reconstructed slice from the reference scan. The two temporal activation cases with different amplitudes lead to different SNR, as identical noise was added to the simulated InI data. The right panel shows the dynamically estimated regularization for the two cases. The dynamically regularized MNE (dMNE) shows significant improvements over the standard MNE in terms of differentiating the baseline from the activation period. Note the different scale in the MNE and dMNE activation time-course plots.*

DISCUSSION

Regularization is an important factor in many applications of parallel acquisition MRI. In this work we propose a time dependent method for adaptive regularization of dynamic magnetic resonance InI. The method is based on a separable Variational Bayesian approximation of the full posterior distribution, with a dynamic model for the regularization. With simulated data, the method shows clear improvements over standard MNE in separating the baseline from the activation period. As a general-purpose algorithm, the proposed method could be adopted to other dynamic imaging applications as well.

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