Coarse-to-fine Iterative Reweighted l_1-norm Compressed Sensing for Dynamic Imaging

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Introduction: Compressed sensing (CS) [1-3] is a method for accelerating acquisitions of sparse/compressible images, HYPR [4] processing is a technique that exploits a composite temporally averaged image to constrain the reconstruction and achieve high acceleration. Inspired by the HYPR use of composite images, we present a modified coarse-to-fine compressed sensing-like reconstruction that uses the composite-like images as (a less constraining) sparsity promoting prior. It is a relaxation of the HYPR- ℓ_0 [5] and iHYPR methods [6].

Theory: The CS reconstruction [1-3] is formulated as ℓ_1-norm minimization. Recent papers [7-8] suggest that direct minimization ℓ_0-norm often results in better reconstructions. One interesting approach is to iteratively reweight the ℓ_1-norm such that it resembles the ℓ_0-norm [8], e.g., $||Wx||_1 \approx ||x||_0$ *for* $w_i = 1/(|x_i|+\mu)$. At each iteration W is set by the result, x, of the previous iteration. Such weighting, W, promotes sparsity since small signal values will be weighted more heavily than larger components in the next iteration. In the same spirit, but in the context of dynamic imaging, weighting the ℓ_1-norm according to a *composite image* (similarly to the composite in HYPR processing) is a way to "inject" the dynamic information to the CS reconstruction.

Methods: Figure 1 describes in detail a coarse-to-fine iterative reweighted ℓ_1-norm CS scheme. It combines the ideas behind HYPR-like methods [4-6] and the CS [3] framework.

Let τ be a scan time interval, initialized to $\tau_0=[0,T]$. F_τ is a (non-uniform) Fourier transform corresponding to the acquisition trajectory at time interval τ, and S *is* a coil sensitivity matrix. Denote ψ as the sparsifying transform, and W is a diagonal weighting matrix. Let x_τ and y_τ be the target image and acquired data at interval τ. The algorithm does the following:

(1) Solve: $||F_\tau S x_\tau - y_\tau||_2+\lambda||W\psi x_\tau||_1$; (2) Stop if τ reaches target temporal resolution; (3) Set weighting W such that $w_i=1/((\psi x_\tau)_i+\mu)$; (4) Split τ into smaller intervals τ'. For each interval repeat steps 1-4 with new weighting, adjust $\lambda=\lambda\sqrt{(\tau'/\tau)}$ according to the new interval and use current x_τ as the initial value.

Results: Data was acquired using a stack of stars radial trajectory during a contrast injection (TR=5.2ms, 620 projections, 72 slices and 8 receivers). The images were reconstructed using the proposed scheme using coarse to fine weighted ℓ_1-norm penalty on the image and spatial finite-differences (TV). Finest temporal resolution was 16 projections (6sec). Other parameters: $\mu=0.1$, *20* CS iterations in each scale solved with SparseMRI [3]. The results compared to CS with SENSE reconstruction from 16 projections. Figure 2 shows superior quality early stage arterial and late stage venus and muscle enhancements in a coronal slice.

Discussion and Conclusions: We presented a method that combines the ideas of HYPR and compressed sensing for extremely high accelerations. Advantages: (1) This method is less susceptible to motion because of refined composite and relaxed constraints compared to HYPR; (2) Sparsity prior can be in any transform domain, not just image space; and (3) It can be used with arbitrary trajectories. Disadvantages: (1) Many parameters need to be adjusted and optimized; and (2) This is a computationally intensive algorithm.

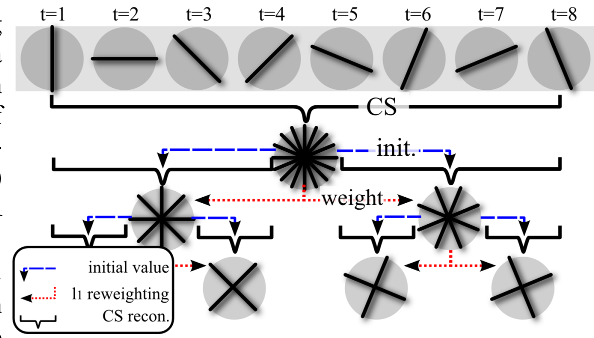

***Figure 1:** Coarse-to-fine iterative reweighted compressed sensing algorithm. At each temporal scale, a "composite" image is reconstructed using a CS reconstruction. The result is used as an initial image for the next finer scale. In addition it is used to generate weighting of l1-norm in the CS reconstruction, promoting sparsity according to the composite. Here a dyadic polar scheme is shown as an example, but overlapping windows and other trajectories are also possible.*

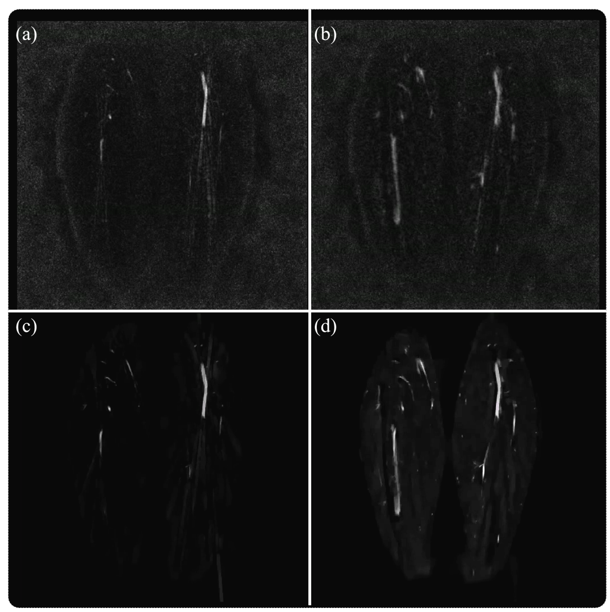

***Figure 2:** Early (left) and late (right) stage post contrast injection reconstructed slice from 16 projections. (a-b) Compressed sensing with SENSE results in blurring and residual artifacts whereas the (c-d) Coarse-to-fine iterative reweighted method recovers a high quality image preserving high resolution features and high temporal resolution.*

References: [1] Cande`s *et al*., IEEE TIT 2006;52(2):489-509 [2] D.L. Donoho, IEEE TIT 2006;52(4):1289-1306; [3] Lustig *et. al* MRM 2007;58(6):1182-95 [4] Mistretta CA *et al*, MRM 2006;55:30-50. [5] Velikina J. *et al* ISMRM'09 pp. 276 [6] O'Halloran RL *et. al*, MRM 2008;59(1):132-9 [7] Trzasko *et. al*, IEEE TMI 2009;28(1):106-21 [8] E.J. Candes, *et. al*, Journal of Fourier Analysis and Appl., 2008;14(5):877-905