# $l_1$-denoised Autocalibrating Parallel Imaging

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**Introduction:** There has been a rising interest in combining parallel imaging (PI) [1,2] with compressed sensing (CS) [3] to achieve high acceleration and reduce the noise amplification that comes from traditional PI reconstruction (e.g. SENSE[1], GRAPPA[2], etc). Existing approaches are: (a) sequentially apply PI and CS [4]; (b) sequentially apply CS and PI [5]; (c) simultaneously apply PI and CS [6-8]. However, noise correlation introduced by autocalibrating PI reconstruction has not been utilized before. In this work, we present a novel sequential approach that consists in first applying GRAPPA followed by CS. We use the fact that a noise covariance matrix of the GRAPPA reconstruction can be constructed from the GRAPPA interpolation kernels. The covariance matrix is used to "intelligently inform" the CS optimization about the confidence levels of the PI reconstruction entries. This results in better CS reconstruction that efficiently suppresses noise and provides high image quality.

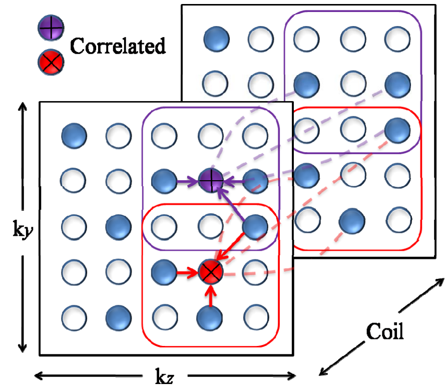


*Figure 1: In GRAPPA, k-space data are correlated after interpolation (solid and dashed lines). Noise correlation due to overlapping of interpolation kernels can also be determined by GRAPPA interpolation weights.*

**Theory and Method:** As shown in Fig. 1, after GRAPPA data synthesis, reconstructed entries are correlated. In presence of noise, the synthesized value for each point and its variance tell us the range of the true value of the point; the covariance matrix tells how the noise is correlated. With this knowledge, CS optimization can then suppress the noise according to its underlying correlation. Let $y$ be the entire acquired $k$-space grid data for all coils, $x_i$ be the entire $k$-space grid data for the $i$th coil, and $G_i$ be the GRAPPA interpolation matrix for the $i$th coil. Data synthesis can then be written as: $x_i = G_i y$. For simplicity, assume the noise for the acquired data is independently identically distributed (i.i.d.) Gaussian noise with variance $\sigma^2$ (the actual noise correlation of the acquired data could be obtained with an extra scan). Post synthesis, the covariance matrix of the noise $\Sigma_i$ for the $i$th coil is: $\Sigma_i = \sigma^2 G_i G_i^H$ . Equipped with this information, we perform a modified CS optimization. Joint sparsity is used as the objective function [5,6,10]:

$$\text{minimize } \Sigma_r(\Sigma_i||w_i(r)||^2)^{1/2}, \text{ subject to } ||\Sigma_i^{-1/2}(\overline{x_i} - x_i)||_2 < \varepsilon,\ \ w_i=\psi F^{-1}\overline{x_i},\ i=1,\ \ldots\ n,$$

where $x_i$ is the GRAPPA reconstruction result, $\overline{x_i}$ is the actual $k$-space data, $F$ is a Fourier transform operator, and $\psi$ is a sparsifying transform operator. It is difficult to calculate the inversion of $\Sigma_i$ directly. Instead, we approximate the inversion of $\Sigma_i$ iteratively while calculating the nonlinear conjugate gradient of the objective function [3]. Furthermore, approximation of $\Sigma_i$ (e.g., only considering the variance) can also save computation cost. Individual coil images are combined using sum-of-squares.

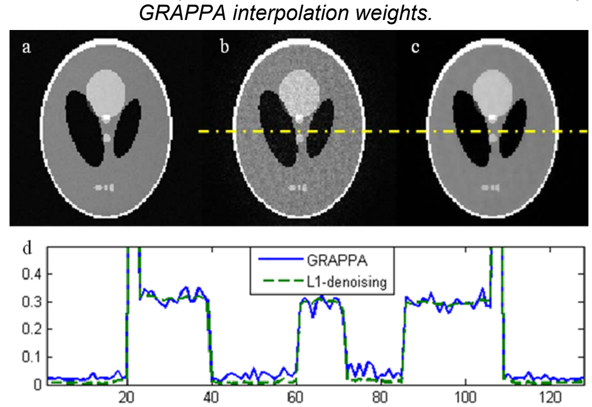


*Figure 2: (a) Original image; (b) 4-fold jittered sampling GRAPPA reconstruction; (c) Reconstruction by proposed method; (d) 1D profiles of the marked line in (b) and (c).*

**Results:** A phantom simulation was performed to demonstrate the proposed method (Fig. 2). Eight-channel Shepp-Logan phantom data set (image size: 128x128) was generated with i.i.d Gaussian noise added separately into each image (SNR=50). Jittered sampling [9] with 2x2 ($k_y$ x $k_z$) undersampling and autocalibrating signals 20x20 was applied. GRAPPA reconstruction was performed with interpolation kernel size 7x7. Total variation (TV) penalty was used for $l_1$ constraints. Similar reconstruction results were achieved by only considering the variance information. However, reconstruction was less accurate when no covariance information was used. Simulation with similar parameters as in the phantom experiment was performed on an 8-channel 3D extremity exam acquired on a 1.5T GE Signa Excite scanner. Reconstruction results of GRAPPA and proposed method with wavelet and TV penalties are shown in Fig. 3. As can be seen, the proposed method efficiently reduced the noise that was amplified during GRAPPA data synthesis.

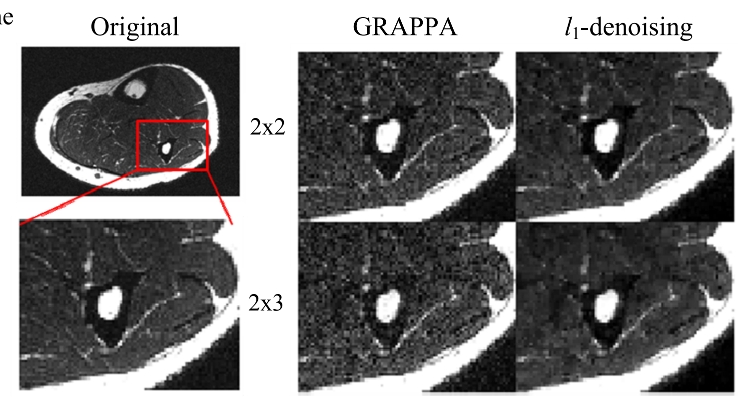


*Figure 3: Simulation by undersampling data from an extremity exam. jittered sampling with 2x2 and 2x3 acceleration was applied. Reconstruction results of GRAPPA and the proposed methods (using variance information only) are shown. Fully sampled data is shown on the left. Noise was efficiently suppressed by the proposed method.*

**Conclusion:** The proposed method sequentially applies GRAPPA and CS. Using the noise correlation obtained during GRAPPA data synthesis, $l_1$-denoising can efficiently suppress noise according to its correlation and improve image quality. Using only variance information achieves similar reconstruction quality and further reduces computation cost.

**References** [1]. Pruessmann, *et al*. MRM 1999; 42(5): 952-962. [2]. Griswold, *et al*. MRM 2002; 47(6): 1202-1210. [3]. Lustig, *et al.* MRM 2007; 58(6): 1182-1195. [4]. Beatty, *et al*. ISMRM'09, p2824. [5]. Liang, *et al.* ISMRM'09, p377. [6]. Lustig, *et al.* ISMRM'09, p379. [7]. Fischer, *et al.* ISMRM'09, p2813. [8]. Otazo, *et al*. ISMRM'09, p378. [9]. Cook, *et al*. ACM TOG 1986; 5(1): 51-72. [10]. Kim, *et al*. ISMRM'09, p382.