Combining nonconvex compressed sensing and GRAPPA using the nullspace method

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Introduction

There has been substantial interest in combining parallel image reconstruction methods like GRAPPA [1] with compressed sensing (CS) [2-4] to improve image quality at higher accelerations. Recent efforts have approached combining these methods either by cascading or iterating GRAPPA-type and CS-type reconstructions [5-6] or by performing joint optimization with multiple objective functions [7]. The proposed method extends CS reconstruction in the nullspace of the acquired data [8] to jointly minimize the ℓ_0-norm sparsity penalty and the ℓ_2-norm deviation from GRAPPA reconstruction. This method approximates the ℓ_0-norm using continuation with a differentiable nonconvex regularizer [9-10]. While this method generalizes to any sparsifying transform, we choose the discrete contourlet sparsifying transform [11] to illustrate its effectiveness in representing brain images. We demonstrate the proposed method reconstructs images at higher accelerations than GRAPPA or CS alone would allow.

Methods

We desire to find full-FOV P-channel k-space data y that minimizes both the sparsity penalty $\|\Psi[C_1,\ldots,C_P]F^{-1}y\|_0$ and the per-channel weighted GRAPPA consistency penalty $\|\mathrm{diag}(C_1,\ldots,C_P)F^{-1}(y-Wd)\|^2$, which is a function of the reduced-FOV acquired k-space data d. Here, Ψ is the sparsifying transform (e.g. contourlet), $C_1,\ldots,C_P$ are the spatial weights used to combine the channel images, F is the discrete Fourier transform (repeated for each channel), and W is the GRAPPA reconstruction operator. To preserve the acquired k-space data and reduce the dimensionality of the optimization problem, this minimization is performed under the hard constraint that $Ky = d$, where K is the sampling matrix for the acquired data. Let x represent the missing k-space data; then y can be expressed in terms of the acquired and missing data: $y = K^{T}d + (I-K)^{T}x$. Here, $(I-K)$ is the sampling matrix for the missing data. This substitution yields the "nullspace method," a joint unconstrained optimization problem in the nullspace of K: minimize $Q(x) = \|\Psi[C_1,\ldots,C_P]F^{-1}(K^{T}d + (I-K)^{T}x)\|_0 + \lambda\|\mathrm{diag}(C_1,\ldots,C_P)F^{-1}((K^{T}-W)d + (I-K)^{T}x)\|^2$, where the tuning parameter $\lambda > 0$ allows us to trade sparsity for consistency with GRAPPA along the Pareto-optimality curve. Since minimizing the ℓ_0-norm is NP-hard, we approximate the ℓ_0-norm using continuation over the sequence $\lim_{\alpha\to\infty} \Sigma_i\ 1\text{-}\exp(-\alpha|x_i|^2)$. A local solution to this optimization problem can be found using an iterative reweighted least squares approach for a given α, and the problem can be repeated for larger α until convergence is achieved. The algorithm is implemented in MATLAB and consists of an iterative reweighted least squares method implemented using the built-in LSQR, which requires two FFTs, two sparsifying transforms, and several sparse matrix-vector multiplications per iteration. The continuation method involves increasing α by a user-set multiplicative factor until the final image converges. The channel weights are estimated from the low-resolution auto-calibration signals (ACS lines) used for training the GRAPPA kernel and are normalized using the sum-of-squares combined image. GRAPPA is performed using overlap-add convolution. The tuning parameter λ is chosen manually based on confidence in the GRAPPA reconstruction. To form a single image, the full FOV k-space data are combined using the estimated channel weights.

Results

We tested the proposed method in 2D on several axial slices from fully-sampled MPRAGE (256x256x176 sagittal slices, 1.0mm resolution) acquired with a 32-channel coil at 3T. After manually cropping the slices tightly to the head, we extracted the central 24 lines in the phase-encode (vertical in Fig. 1) direction to use as our ACS lines for estimating channel combination weights and training our GRAPPA kernel. We then uniformly downsampled k-space by a factor of 5 in the phase-encode direction, keeping the ACS lines, and reconstructed the full-FOV image using GRAPPA, the nullspace CS method [8], and the proposed method. In the nullspace CS and proposed methods, the contourlet sparsifying transform was used. The proposed method was evaluated for values of λ spaced logarithmically between 10^{-10} (more sparsity) and 10^{9} (less sparsity). Fig. 1 portrays the proposed nullspace CS+GRAPPA alongside GRAPPA and nullspace CS for two different slices (labeled A and B). When compared to GRAPPA, the proposed method reduced the noise in the brain and visibly mitigated the aliasing. The PSNR (peak-signal-to-noise ratio) increased by 2.5 dB and 2.3 dB relative to the GRAPPA results for examples 1A and 1B, respectively. In addition, fine edges, such as those in the circled regions, were preserved better by the proposed method than by CS alone.

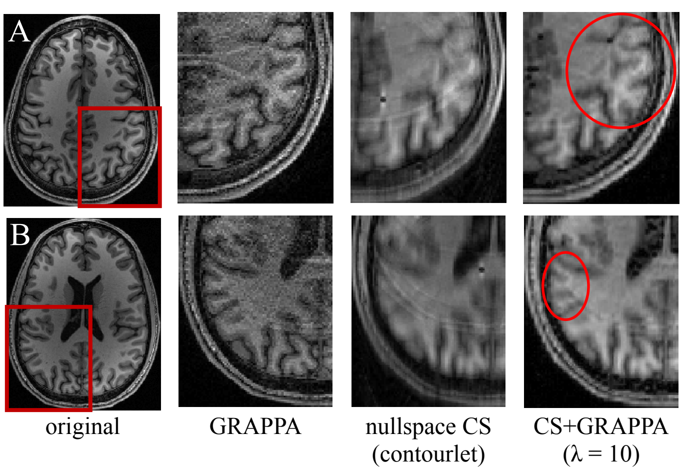

Fig. 1: Reconstruction of two uniformly under-sampled brain image slices A and B using GRAPPA, nullspace CS, and the proposed nullspace CS+GRAPPA method.

Conclusions

The proposed method provides a convenient framework for combining CS and GRAPPA and enforces consistency with the acquired data. This method can be easily generalized to non-uniform k-space sampling patterns. Future work will focus on extending this method to variable-density patterns in k-space and investigating the performance of different approximations to the ℓ_0-norm in this framework.

References

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