A new approach to incorporate image prior estimate in compressed sensing

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Introduction The success of CS [1] is limited by the sparsity of the underlying image. In [2], we proposed to incorporate a data sorting process in the CS recovery so that the underlying image can be recovered in an alternative form that features a higher level of sparsity. We here show that performing a data sorting effectively incorporates a prior image estimate in CS reconstruction.

Theory In case that a sorting R is performed on the underlying image $\mathbf{f}$ in the CS recovery, we attempt to gain an estimate of the sorted image $\mathbf{g}$ instead, and the final image estimate $\hat{\mathbf{f}}$ is obtained by applying an reverse sorting (denoted as R^{-1}) to the estimate $\hat{\mathbf{g}}$:

$$\mathbf{f} \xrightarrow{R} \mathbf{g}, \;\; \hat{\mathbf{g}} = \mathrm{argmin}_{\mathbf{g}}(\parallel \mathbf{y} - \mathrm{W}_R \mathrm{g} \parallel_2 + \; \alpha \parallel \Phi \mathrm{g} \parallel_1) \;, \;\; \hat{\mathbf{g}} \xrightarrow{R^{-1}} \hat{\mathbf{f}} \qquad (1)$$

where $\mathbf{y}$ and W_R respectively denotes the acquired data set and the Fourier encoding matrix whose columns are rearranged to reflect the sorting process R; Φ is the sparsifying transform used. In practice the perfect sorting order R is unknown as the image $\mathbf{f}$ is yet to be recovered, instead an approximate sorting order can be obtained by sorting a prior estimate of the image. In the following, we use a 1D signal (left, Fig.1.a) to illustrate the effects of data sorting.

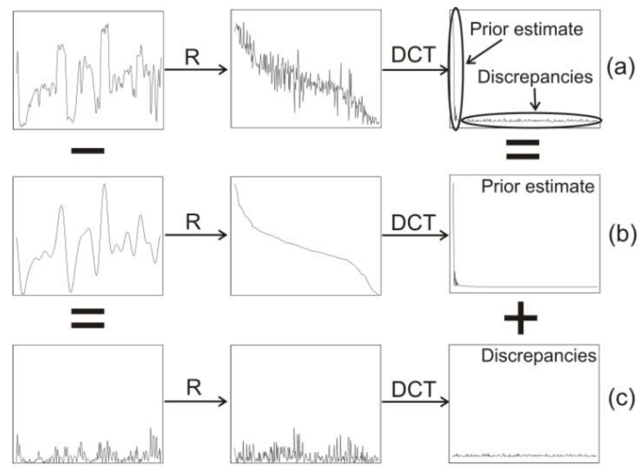

Figure 1: Effects of data sorting on a 1D signal using a sorting order obtained from a low resolution approximation with a DCT used.

In Fig. 1.b, a low resolution approximation (the prior estimate) of the signal (left, Fig.1.b) is sorted to form a monotonic variation (middle, Fig.1.b) with a sorting order R, who has a very sparse DCT transform (right, Fig.1.b). Then this sorting order is used to sort the original signal (left, Fig.1.a), which leads to an imperfectly sorted signal (middle, Fig.1.a). Now, it is the transform of this imperfectly sorted signal (right, Fig.1.a) to be recovered using Eq.(1).

The actual signal can be decomposed into two parts: the prior estimate and the discrepancies between the estimate and the signal (left, Fig.1.c). Likewise, the transform of the imperfectly sorted signal under sorting order R is consisted of two parts: the strong components attributed by the prior estimate and the weak components attributed by the discrepancies. By the nature of the CS recovery, the strong components would be recovered with high fidelity, which means that the prior estimate is intrinsically embedded in the reconstruction of the imperfectly sorted signal (right, Fig.1.a). Thus the use of a sorting order obtained from a prior estimate allows the prior estimate to be incorporated in the CS reconstruction.

Method A 2D T2-weighted axial brain slice was obtained (256×256) using a 1.5T GE scanner equipped with an 8-channel head coil. The following experiment was designed to investigate the incorporation of an image prior estimate via a data sorting. Firstly, a pseudo-random sampling mask at acceleration factor of 4 with a uniform sampling density was created; a low resolution approximation of the image was obtained using fully sampled 32 lines at k-space center and used as a prior image estimate for obtaining the sorting order. Three types of CS reconstructions were performed: (a) with the uniform-density sampling pattern; (b) with the same sampling pattern but the central 32 lines are fully sampled; (c) with the same sampling pattern but incorporates a data sorting order obtained from the low resolution image prior estimate. By the nature of CS recovery, (a) is not likely to lead to a successful reconstruction due to the lack of low frequency components, and the missing low frequency information is incorporated in (b) and (c) in different ways : either directly measured or via a data sorting.

Results and discussion The different CS reconstruction outcome are shown in Fig.2. In Fig.2.a, it is seen that the uniform-density sampling pattern leads to an unsuccessful CS recovery as expected (Fig.2.a). In Fig.2.b where the low frequency components are measured, a substantially improved recovery is received. In Fig.2.c, the data sorting effectively incorporates the low resolution approximation in the CS recovery and compensated for the missing low frequency information. It is surprising to observe that even better recovery of image fine features is received in Fig.2.c (as arrowed). This is conjectured to be because data sorting disrupts the original form of CS reconstruction artifacts (loss of image details and contrast [1]), and is to be further investigated.

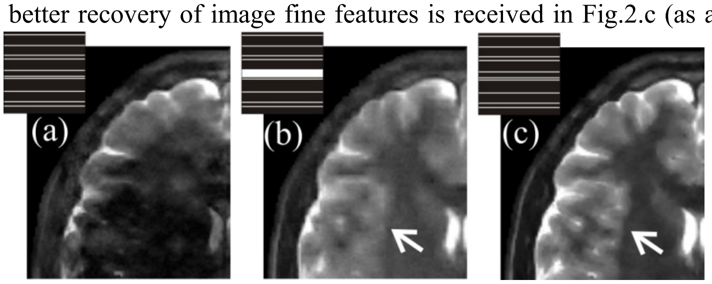

Figure 2: different CS reconstructions with sampling patterns used as shown. In (c), a data sorting is incorporated to compensate for the lack of low frequency measurements.

Conclusion We have shown both in theory and using experiment that performing a data sorting effectively incorporates a prior image estimate in the CS reconstruction. This may lead to many interesting applications where a good prior estimate is available, such as in time-resolved dynamic imaging.

Reference [1] Lustig M, et al. MRM, 2007
[2] Wu B, et al. Proc. ISMRM, 4595, 2009.