# A Hybrid L0-L1 Minimization Algorithm for Compressed Sensing MRI

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## INTRODUCTION

Both $L_1$ minimization [1] and homotopic $L_0$ minimization [2] techniques have shown success in compressed-sensing MRI reconstruction using reduced *k*-space data. $L_1$ minimization algorithm is known to usually shrink the magnitude of reconstructions especially for larger coefficients [1, 3] and non-convex penalty used in homotopic $L_0$ minimization is advocated to replace $L_1$ penalty [3]. However, homotopic $L_0$ minimization only finds local minimum which may not be sufficiently robust when the signal is not strictly sparse but also has small elements after a sparsifying transform or the measurements are contaminated by noise [4]. Since practical MR images are never strictly sparse after a transform, it is desirable to estimate both large and small coefficients more accurately. In this abstract, we propose a homotopic $L_0$-$L_1$ hybrid minimization algorithm to combine the benefits of both $L_1$ and homotopic $L_0$ minimization algorithms for MRI. The proposed algorithm minimizes the $L_0$ quasi-norm of large transform coefficients but the $L_1$ norm of small transform coefficients for the image to be reconstructed. The experimental results show the proposed algorithm outperforms either homotopic $L_0$ or $L_1$ minimization when the same reduction factor is used.

## THEORY AND METHOD

The proposed algorithm is formulated as solving the following optimization problem: $\min_{\mathbf{x}} \|\mathbf{x}\|_{0/1}$ s.t. $\mathbf{y} = \mathbf{\Phi x}$ (1), where $\|\mathbf{x}\|_{0/1} = \sum_i f(x_i)$ is a hybrid $L_0$-$L_1$ quasi-norm with $f(x_i) = \begin{cases} |x_i| & |x_i| < \tau \\ 1 & |x_i| \geq \tau \end{cases}$, and $\tau$ is the threshold between the choice of $L_1$ norm for small elements and $L_0$ quasi-norm for large elements. Similar to the homotopic $L_0$ minimization algorithms, the desired minimization problem in (1) is approximately solved by a sequence of $L_1$ minimization problems minimizing $\sum_i \rho(x_i, \varepsilon)$. In this case, the function $\rho(x, \varepsilon)$ is chosen to be concave and approach the desired hybrid $L_0$-$L_1$ quasi-norm function as a sequence limit: $\lim_{\varepsilon \to 0} \rho(x, \varepsilon) = \begin{cases} a|x_i|/\tau & |x_i| < \tau \\ \dfrac{\big||x_i| - b\big|}{\big||x_i| - b\big| + \varepsilon} & |x_i| \geq \tau \end{cases}$. Constants $a$ and $b$ are chosen to make the function continuous and differentiable at $|x| = \tau$. Figure 1 shows how the function $\rho(x, \varepsilon)$ approaches the desired hybrid quasi-norm when $\varepsilon$ approaches zero for $\tau = 1$. In our choice for the cost function, strict concavity is the key to assuring solution uniqueness for the compressed sensing reconstruction problem [2]. Although a similar hybrid $L_0$-$L_1$ quasi-norm function has been applied to the finite difference of an image as a prior for SENSE regularization [5], the proposed hybrid quasi-norm is used in the context of compressed sensing, where random undersampling is used to construct an underdetermined linear equation and the hybrid quasi-norm is applied to the coefficients of any sparse transforms. Simulation was carried out to compare the proposed algorithm with $L_1$ and homotopic $L_0$ minimizations. Iteratively reweighted $L_1$ minimization [6] was used among existing homotopic $L_0$ minimization algorithms. An undersampled radial trajectory was used to generate the simulated *k*-space data and Gaussian noise was then added on the data. Two parameters $\alpha$ and $\beta$ for measuring noise suppression and edge preservation [7] were calculated, which take larger values when image quality improves.

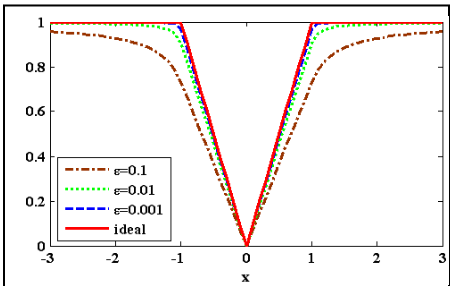


**Figure 1:** Plots of $\rho(x, \varepsilon)$ as $\varepsilon$ approaches zero with $\tau = 1$.

## RESULTS AND DISCUSSION:

Figure 2 shows the plot of sorted magnitude of a 256 × 256 phantom. It shows the phantom consists of some large elements which decay rapidly and many small elements. Figure 3 shows the reconstruction results for the phantom simulation. The reduction factor is 3.3 and the SNR is 20dB. Identity transform was used as the sparsifying transform. Parameter $\tau$ was chosen to be 0.55 which is the "corner" of the plot in Figure 2. The algorithm and corresponding $\alpha/\beta$ (%) are labeled on the top and bottom of each reconstructed image. In addition, the corresponding "comb" region in the phantom was zoomed to reveal details. It is seen homotopic $L_0$-$L_1$ hybrid minimization algorithm outperforms the other two algorithms both visually and in term of $\alpha/\beta$ values. It suggests the proposed algorithm better preserves details and suppresses noise and artifacts. Future work will investigate optimal choice of parameters $\tau$ in absence of prior knowledge.

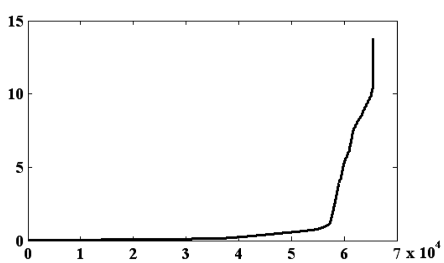


**Figure 2:** Plot of sorted magnitude of phantom.

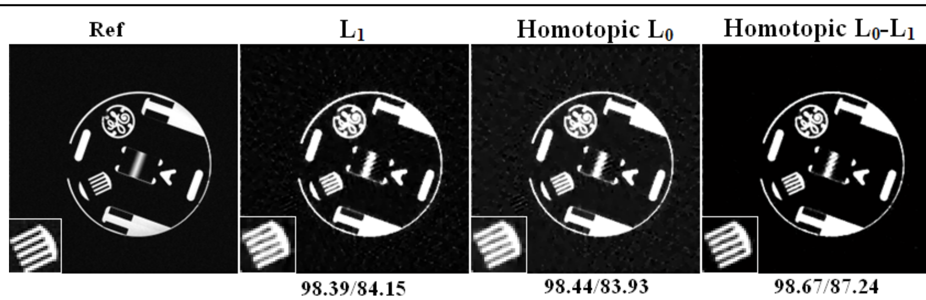


**Figure 3:** Phantom reconstructions using three competing algorithms.

## CONCLUSION

A novel algorithm is proposed to integrate $L_1$ and homotopic $L_0$ minimizations for compressed-sensing MRI reconstruction. The results show that the proposed algorithm can outperform the $L_1$ and homotopic $L_0$ minimization algorithms in preserving details and suppressing noise and artifacts.

## REFERENCES

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