Optimal single-shot k-space trajectory design for non-Cartesian sparse MRI

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Introduction: Sparse MRI [1] is able to reduce the acquisition time and raw data size by significantly undersampling the *k*-space. However, the issue on how to design an optimal *k*-space trajectory on sparse *k*-space is technically challenging and has not been addressed. In this work, we use graphic theory to design an optimized non-Cartesian *k*-trajectory which can further decrease the acquisition time. The *k*-space is firstly sampled using the Monto-Carlo sampling schemes [1] and then an optimal single-shot *k*-trajectory traveling through all these samples is designed using the simulated annealing (SA) algorithm [2]. Finally the corresponding gradient waveforms are designed using the time-optimal gradient algorithm [3]. For comparison, conventional single-shot Cartesian EPI trajectory and spiral trajectory traveling through all the same *k*-space samples as well as their corresponding gradients are designed.

Theory and method: The *k*-space sampling followed Monte-Carlo incoherent sampling strategy used in Ref. [1]. The SA was then used to design an optimal single-shot *k*-space trajectory traveling through all these samples. The *k*-space trajectory was initiated by connecting all the samples using a continuous curve and the cost function of SA was defined as the curve length:

$$f = \sum_{j=0}^{N_s-1} \left| k_{j+1} - k_j \right| \quad (1),$$

where N_s is the number of *k*-space samples and the k_j denotes the *j*th sample. By randomly changing the sequence of two samples, the trajectory and its length were changed. Here, the cost function values before and after changing are denoted by f_1 and f_2 respectively, and the Boltzmann probability distribution was calculated by:

$$p = \exp((f_1 - f_2)/k_B T) \quad (2),$$

where T is the temperature of the system and k_B is the Boltzmann's constant that relates the temperature to the cost function. Whether to accept the new trajectory was determined by comparing a random number *r* and the Boltzmann probability *p*: if $r<p$ the new trajectory would be accepted otherwise rejected. It is noticed that if the length of the new trajectory f_2 was shorter than the previous one f_1, the new trajectory would always be accepted due to *p* was always greater than 1. If f_2 was larger than f_1, there was still some possibility for f_2 to be accepted. By utilizing this general annealing scheme, which usually took a downhill step while sometimes took an uphill step, the SA had the ability to break away from local minimum and to search the whole space for a global optimal solution. The optimization flowchart is shown in figure 1.

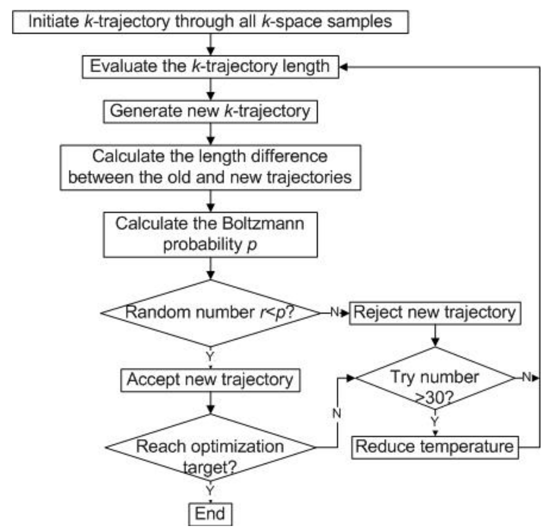

Fig. 1 Flowchart of optimizing *k*-trajectory by SA.

After designing the optimal sparse *k*-space trajectory, the corresponding gradient waveforms were designed using the time-optimal gradient method [3] which was able to design gradients with minimum time for arbitrary trajectories. For comparison, Cartesian EPI and spiral trajectories passing through all the same *k*-space samples were designed and their corresponding gradients were also designed using the time-optimal gradient method. The maximum k-space extension was 5 cm^{-1}. The gradient maximum amplitude and slew rate were 4Gauss/cm and 15Gauss/cm/ms. Matlab7.6 (Mathworks Co.) was used for all calculations.

Results: Figure 2 shows the design results of three different *k*-space trajectories and their gradients. On the left column are the *k*-space trajectory plots. The red stars denote the sparse *k*-space samplings and the blue curves denote different trajectories. It can be seen that to travel through the same *k*-space samples the optimal trajectory designed by SA has the shortest length. From the gradient waveforms on the right column, it is shown that the gradients length of optimal *k*-trajectory is 33 ms, while those of the spiral and Cartesian EPI are 54 ms and 72 ms respectively. All the gradient waveforms satisfy the maximum gradient and slew rate requirement.

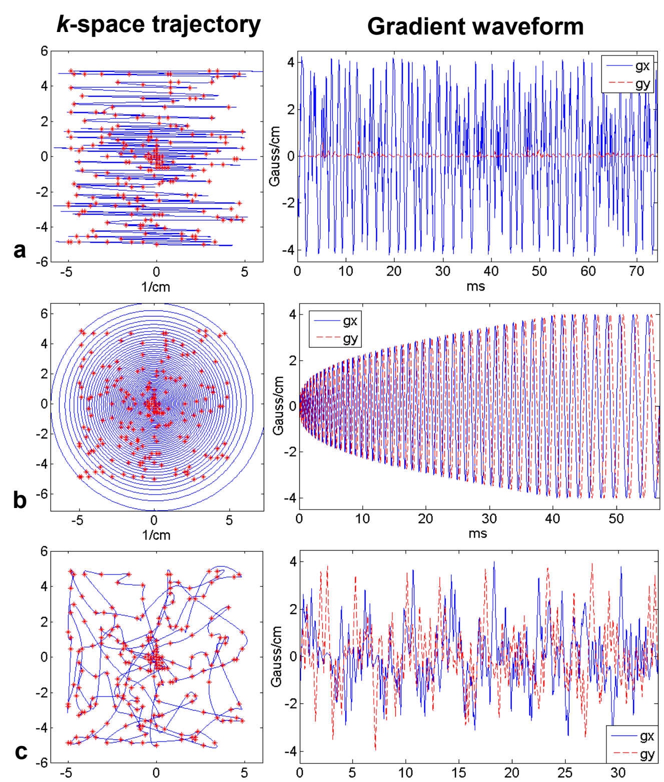

Fig. 2 Comparison between three designs: (a) Cartesian EPI; (b) variable density spiral; (c) optimal sparse *k*-traj. The left column is the *k*-space: the red the stars denote the sparse sampling and the blue curves are the *k*-trajectories. Their corresponding gradients are on the right column.

Conclusions and discussions: In this work, a novel *k*-space trajectory design strategy has been developed for sparse MRI. By using the SA, an optimal *k*-trajectory traveling through all the sparse *k*-space samples can be obtained, further shortening the acquisition time. It is also noticed that rapid changing of the gradient waveforms has a high performance requirement for the gradient coils.

References: [1] Lustig M, et al, Magn Reson Med 2007; 58: 1182-1195. [2] Kirkpatrick S, et al, Science 1983, 220: 671-680. [3] Lustig M, et al, IEEE Trans Med Imag 2008; 27: 866-873.

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