Total Generalized Variation (TGV) for MRI

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Introduction: Total Variation (TV) based strategies, which were originally designed for denoising of images [1], have recently gained wide interest for many MRI applications such as regularization in image reconstruction and deconvolution. TV models have the main benefit that they are very well suited to remove random fluctuations and noise-like artifacts from sub-sampled scans (radial or random), while preserving the edges in the image. However, the assumption of TV is that the images consist of regions which are piecewise constant. Inhomogeneities of the coil sensitivities and the exiting B_1 field often violate this assumption in practical MRI examinations. Additionally, there are also natural gradually changing signal intensities in the investigated anatomies. In all these situations TV leads to staircasing artifacts and results in patchy images, which appear unnatural. This paper introduces the new concept of Total Generalized Variation (TGV) as a penalty term for MRI problems. This mathematical theory has recently been developed [2], and while it is equivalent to TV in terms of edge preservation and noise removal, it can also be applied in imaging situations where the assumption that the image is piecewise constant is not valid. As a result, the application of TGV in MR imaging is far less restrictive. It is shown in this work that TGV can be applied for image denoising and for regularization of iterative image reconstruction of undersampled radial data sets from multi array coils. It yields results that have smooth signal intensity changes and preserved edges.

Methods: The notion of k-th order TGV bases on taking the relevant information from the first k derivatives of an image into account. It is a generalization in the sense that TGV_1 is equivalent to TV which only incorporates first-order derivative information. In this work, only TGV_2 is discussed and it is shown that TGV_2 constitutes a suitable regularization for piecewise smooth MR images. One of it's main features is the absense of the staircasing effect for data violating the piecewise constance condition. Mathematical details of the functional and proofs can be found in [2], but an intuitive explanation of the reason for this is as follows:

Formally, it holds that $TGV_2(u) = \inf_v \alpha_1 \int_\Omega |\nabla u - v| dx + \alpha_0 \int_\Omega \left| \frac{1}{2}\left(\nabla v + \nabla v^T\right) \right| dx$ where v is a smooth vector field and the second term its symmetrized derivative. In smooth regions of u, an optimal v locally approximates the derivative ∇u, resulting in the local penalization of the second derivative $\nabla^2 u$. In the neighborhood of edges, v is approximately zero since $\nabla^2 u$ is much larger than ∇u. This, in turn, leads to a penalization of ∇u. In total, natural looking piecewise smooth images are penalized less than staircase images and therefore preferred in TGV_2-based minimization problems. This is shown in **Fig. 1**, which illustrates the absense of staircasing artifacts in TGV_2 in contrast to conventional TV for denoising of a numerical ramp image.

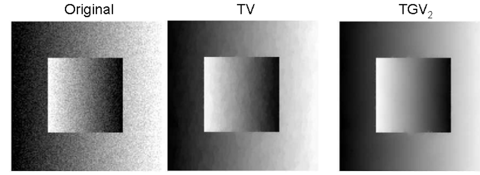


Fig. 1: Illustration of TV (middle) and TGV2 (right) denoising of a numerical ramp image.

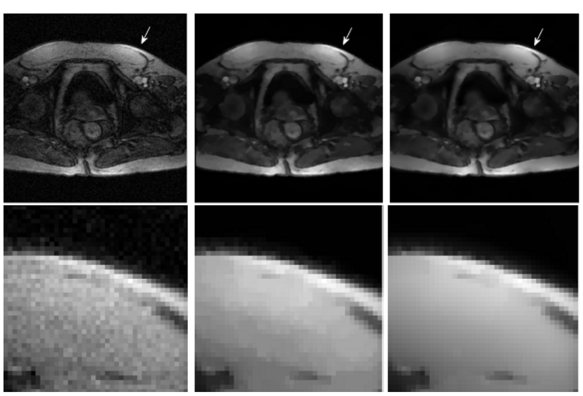
The application of TGV for image denoising is of the form $\min_u \frac{1}{2}\int_\Omega (u-f)^2 + TGV_2(u)$ for a noisy image *f*. A contrast enhanced T1 weighted clinical measurement of the prostate was performed with a 3D gradient echo sequence. All measurements were performed on a clinical 3T system (Siemens TIM Trio, Erlangen, Germany) and written informed consent was obtained from all subjects prior to the examinations. Sequence parameters were TR/TE=3.3/1.1ms, FA=15°, matrix size 256x256, 20 slices, slice thickness 4mm, in plane resolution 0.85x0.85mm. The measurements showed severe signal inhomogeneities due to the exiting b1 field [4] which violated the assumption that the images consist of piecewise constant areas.

For iterative image reconstruction of undersampled radial data from multiple coils, we extended the approach proposed in [3] with an integration of a TGV_2 constraint. Undersampled T2 weighted radial spin echo measurements of the human brain were performed using a receive only 12 channel head coil. Sequence parameters were: TR/TE=2500/50ms, matrix size 256x256, slice thickness 2mm, in plane resolution 0.78x0.78mm.

Results: **Fig. 2** shows the results from the denoising experiments. While TV and TGV_2 have the same denoising behaviour with sharp edges remaining in the image, TV results show significant staircasing artifacts in regions where inhomogeneities violate the assumption of piecewise constant images. **Figs. 3** and **4** compare the reconstructions from undersampled radial imaging. Reconstructions from 48, 32 and 24 projections are displayed. It is not surprising that conventional NUFFT reconstructions show streaking artifacts which get worse as the number of projections is reduced. While both TV and TGV_2 reconstructions eliminate streaking artifacts efficiently, TGV_2 results show a less blocky, more natural appearance.

Discussion: This work presents TGV for MR imaging, a new mathematical framework which is an extension of the TV method. While it shares the existing desireable features of TV, it has additional benefits, which allow using TGV in MR imaging situations where TV fails because of its model assumptions being violated. Additionally, due to the absence of staircasing artifacts in TGV, even in the case of homogeneous images, it delivers images that follow much better the underlying signal changes. It is possible to use TGV in all applications where TV is currently applied. The denoising and image reconstruction experiments of this work showed clearly improved image quality over conventional TV. TGV is based on a solid mathematical theory within the framework of convex optimization, which ensures that numerical implementations are usually straightforward as a variety of numerical algorithms already exist for these types of problems. Concerning additional applications, it is worth noting that it is possible to use TGV as a regularizer that can be applied directly to individual channel array coil data. Thus, it is possible to use TGV in combination with all parallel imaging reconstruction methods that first generate individual coil images, which are then combined to a single sum of squares image in the last step, like GRAPPA. Another promising application for TGV would be diffusion tensor imaging. Currently, TGV is only defined for scalar functions, but as symmetric tensor fields are inherent in the definition, it can be easily extended to higher order tensor fields and could therefore be used as a regularizer to reconstruct the diffusion tensor.

Fig. 2: Image denoising: Original image (left), TV (middle) and TGV_2 (right). The bottom row shows magnified views from a region with severe signal inhomogeneities.



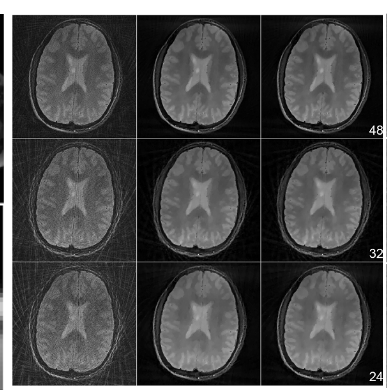


Fig. 3: TGV_2 Image reconstruction of undersampled radial data with 48, 32 and 24 spokes. Conventional NUFFT reconstruction (left), TV (middle) and TGV_2 (right).

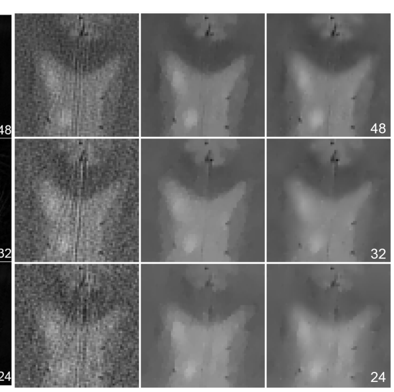


Fig. 4: Magnified views from Fig. 3. Conventional NUFFT reconstruction (left), TV (middle) and TGV_2 (right).

References: [1] Rudin et al. Phys. D, 60(1-4) 259-268, 1992 , [2] Bredies et al., SFB-Report 2009-038, August 2009, [3] Block et al., MRM 57: 1086-1098 (2007), [4] Merwa et al. Proc. ISMRM 2009 p4363