Partial Fourier Compressed Sensing

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Introduction: Partial k-space sampling is often used to reduce the acquisition time (half-Fourier imaging) or shorten the echo time (partial echo imaging) [1]. In the striving to minimize imaging time, even higher dimensional partial Fourier sampling has been proposed, measuring a reduced k-space in two dimensions [2]. However, with such undersampling there is always a part of k-space that cannot be recovered. Combining partial Fourier imaging with other constrained reconstruction methods could improve the reconstruction and yield further acceleration. This has been shown in several papers for a combined parallel imaging-partial Fourier reconstruction [3-5]. In this work, we consider the combination of partial Fourier imaging with compressed sensing (CS) [6-8] and further extend the reconstruction to multi-coil acquisition.

Theory: In partial Fourier imaging a little more than half of k-space is acquired. The image phase is estimated from the fully sampled low resolution data, and the conjugate symmetry of k-space is used to reconstruct a real image. Compressed sensing on the other hand exploits the image sparsity in a given transform domain and requires incoherent sampling. We consider a combined partial k-space compressed sensing reconstruction by constructing a hybrid randomized partial k-space sampling and applying both phase and sparsity constraints in the reconstruction. The two considered sampling patterns are shown in Fig. 1. The central part of k-space is fully sampled and used to estimate the image phase. Alternatively, the phase could be obtained from a calibration scan. The rest of k-space is undersampled according to Poisson disk sampling [9] with coverage corresponding to half-Fourier and partial echo sampling. Poisson disk sampling has been also shown to be beneficial for parallel imaging, since it is much more regular than random sampling for instance [9].

Fig. 1 Randomly undersampled (a) half-Fourier and (b) partial echo sampling pattern. Sampled k-space positions are given in white.

The reconstruction is based on projection onto convex sets (POCS) and is similar to the approach proposed in [10]. Denoting the measured k-space data with y, the image estimate at iteration i with x_i, the sparsifying transform with Ψ and the phase estimate with ϕ, the reconstruction algorithm is given by:

Initialize $x_0 = 0,\ y_0 = y$

1) $x_i = \mathcal{F}^{-1}\{y_i\}$
2) $x_i = \Psi^{-1}(ST(\Psi x_i, \lambda))$
3) $x_i = |x_i| e^{i\phi}$
4) $y_i = \mathcal{F}\{x_i\};\ y_i|_{acq} = y$
5) Repeat until convergence

The soft thresholding operation in 2) is defined as $ST(w,\lambda) = \frac{w}{|w|}(|w| - \lambda)_+$. The parameter λ can be determined automatically using the approach described in [11]. The algorithm is easily extended to multi-coil reconstruction by replacing the Fourier transform with the SENSE encoding matrix $E = \mathcal{F}S$, and the inverse Fourier Transform with its pseudo-inverse $E^+ = (E^H E)^{-1} E^H$.

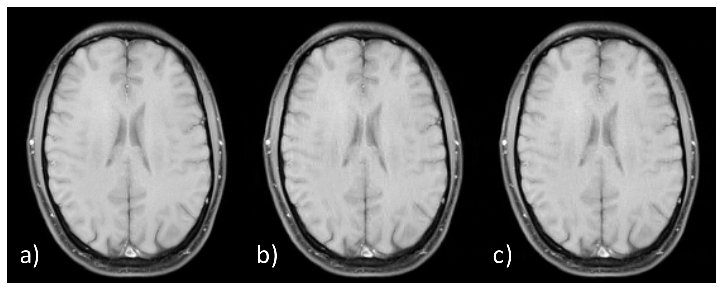

Fig. 2 Partial Fourier CS reconstruction in the brain. a) Full sampling b) undersampled half Fourier R = 2.5, RMSE = 0.0415 c) undersampled partial echo R = 3.1, RMSE = 0.0462 .

Methods: Two brain data sets were acquired on a 1.5T clinical scanner (Philips Healthcare, Best, The Netherlands) with the following parameters: 1) TE = 5ms, TR = 1000ms, FOV = 250mm , 256x256 matrix and 2) 8 channel head coil TE = 20ms TR = 500ms, FOV = 250mm, 256x256 matrix. The data were retrospectively undersampled according to the sampling patterns shown in Fig.1. with the central 40 lines fully sampled in both cases. The reconstruction algorithm was implemented in Matlab (MathWorks). Images were reconstructed according to the algorithm described above applying a collection of sparse bases (Daubechies 4 and Haar wavelets) [12] in step 2 of the algorithm.

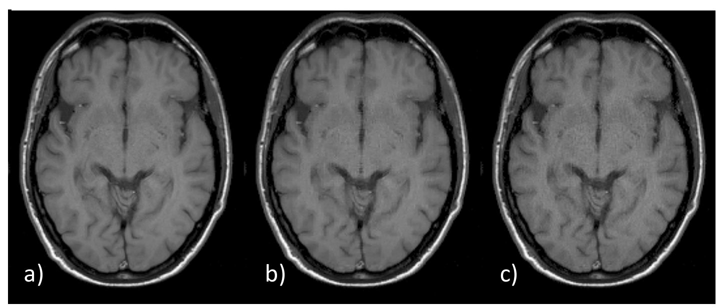

Figure 3 Multi-coil brain data. a) Full sampling b) undersampled half Fourier R = 3.46, RMSE = 0.0541 c) undersampled partial echo R = 4.0 RMSE = 0.0548.

Results and Discussion: Figures 2 and 3 show images obtained with the Nyquist sampled data and with the two randomized partial Fourier undersampling patterns for single and multi-coil reconstruction, respectively. The data reduction factors and the normalized RMS errors are given in the figure captions. The reconstruction time (40 iterations) was 4s for single coil and 28s for the coil array (2.2 GHz CPU, 2GB memory). Good image quality was obtained at higher reduction factors with the multi-coil reconstruction. In the optimal case, the reconstruction will benefit from all three data reduction techniques. The achievable acceleration factor depends on several factors associated with each of the individual techniques such as the amount of phase variations, the signal sparsity, and coil geometry.

Conclusion: We have illustrated that partial Fourier imaging can be combined efficiently with CS reconstruction, which could be interesting for a couple of applications. The POCS-based reconstruction is time efficient and easy to implement. The extension to parallel imaging is straightforward and can lead to further improvement, which comes at the cost of prolonged reconstruction time.

References: [1] Feinberg D et al. Radiology, 1986 161: 527-531; [2] Xu and Haacke JMRI 2001 14:628-635 ;[3] Bydder M and Robson M, MRM 2005 53:1393-1401; [4] Willig-Onwuachi JD et al., JMR 2005 2:187-198:19; [5] Lew C MRM 2007 58:910-921 ; [6] Candes E et al, IEEE Tran Info Theo 2006 52: 489-509;[7] Donoho D, IEEE Tran Info Theo 2006 52: 1289-1306; [8] Lustig M et al, MRM 2007, 1182-1195; [9] Lustig et al, Proc ISMRM 2009: 379; [10] Thüring T et al, Proc ESMRMB 2009: 31 ; [11] Maleki A and Donoho D Proc SPARS 2009; [12] Bilgin et al. Proc ISMRM 2009: 381;