The influence of various adaptive radial undersampling schemes on compressed-sensing L1-regularized reconstruction

R. W-C. Chan[1], E. A. Ramsay[2], and D. B. Plewes[2]
[1]Medical Biophysics, University of Toronto, Toronto, ON, Canada, [2]Imaging Research, University of Toronto, Toronto, ON, Canada

Introduction

Adaptive imaging allows multiple image sets, each having a different spatial-temporal balance, to be retrospectively reconstructed from the same dataset. High temporal resolution image sets from radial sampling schemes are typically undersampled, and suffer from streak artifacts that degrade image quality. It has been shown that a compressed sensing (CS) L1-penalized reconstruction can be used reduce these streak artifacts [1-3]. However, it remains unclear which radial sampling pattern is optimal for CS reconstruction. Here, we compare the effects of 3 adaptive sampling schemes (golden angle [4-5], bit-reversed, and random radial sampling scheme) on the ability of CS reconstruction to reduce streak artifacts, at various spatiotemporal resolutions.

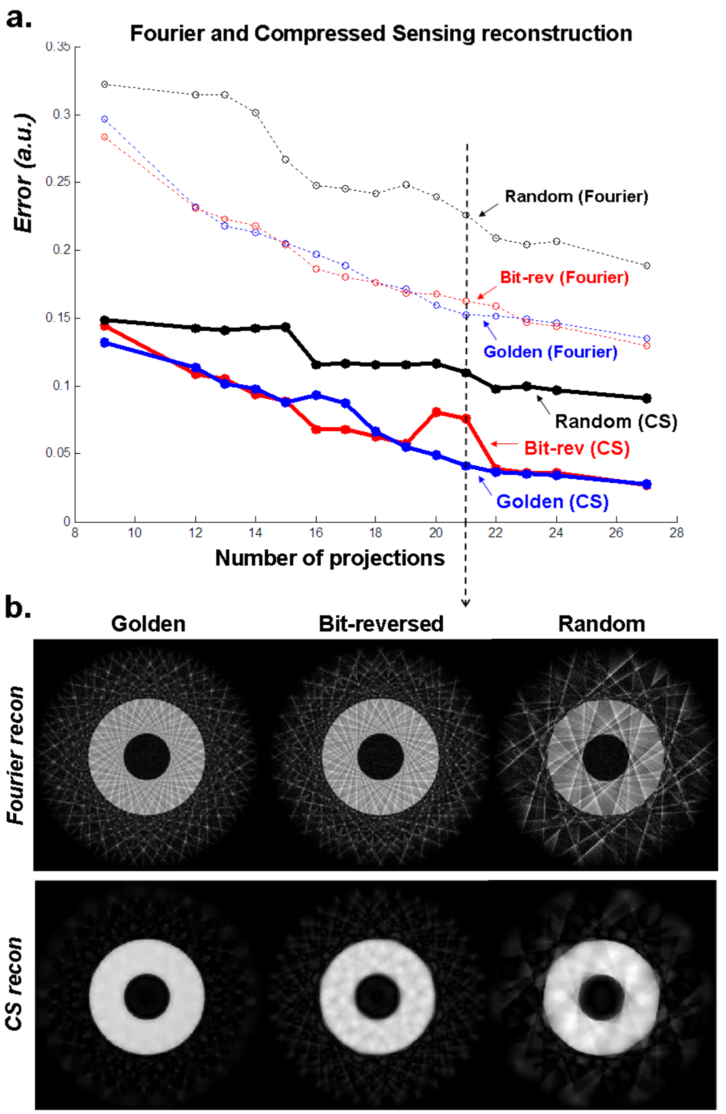

Figure 1: a) The error in Fourier and CS reconstruction is shown for 3 sampling schemes (golden, bit-reversed, and random) for a range of projection numbers. b) Fourier and CS images are shown for 21 projections, for each sampling scheme.

Methods

Simulations: A circularly symmetric object was used in the simulations to isolate the effect of relative angle-spacings in a given spoke-set by eliminating the dependence on the actual k-space locations of the spokes. The analytic k-space of a ring-shaped object (having uniform signal intensity) was sampled, using 3 radial sampling schemes: **i) Golden-angle:** 2D radial projections are successively are incremented by the golden angle ($(\sqrt{5}-1)/2 * 180° \approx 111.25°$) [4-5]. No subset is perfectly evenly-spaced, but spokes are relatively well-distributed for every resolution. **ii) Bit-reversal:** This method generates evenly-spaced projections only at 2^n spokes, where n is an integer. For other resolutions, the spokes are unevenly distributed. **iii) Random:** Projections having random angles (between 0 and 180°) were used for sampling.

Image Reconstruction: For a range of spoke-numbers and for each sampling scheme, images were reconstructed with both Fourier and CS reconstruction. In CS reconstruction, an L1-penalized non-linear conjugate gradient algorithm (Lustig's "SparseMRI" Matlab code [3]) was used to minimize the following cost function: $C(x) = \|F \cdot W^H \cdot x - y\|^2 + \lambda_1 \cdot |x|_1 + \lambda_2 \cdot TV(W^H \cdot x)$, where x is the wavelet image, y is the k-space data, W is the 2D wavelet transform, TV is the total variation operator, F is the Fourier operator, and superscript H denotes the adjoint operator. The regularization parameters used were $\lambda_1=0.02$ and $\lambda_2=0.02$. Gaussian random noise was added to all images (to produce an SNR~30 in the Fourier image reconstructed with enough spokes to satisfy the Nyquist criterion). For each reconstructed image, the ***degree of error*** in the CS reconstruction was characterized by the standard deviation of the pixel intensities over the object, which was scaled to have a mean signal intensity of 1.

Results

Figure 1a shows the degree of error in images after Fourier and CS reconstruction. This is illustrated for a range of spoke numbers, for each sampling scheme. In all cases, the CS reconstruction showed reduced reconstruction error compared to the Fourier reconstruction. Differences among the sampling schemes are evident in CS-reconstructed images at intermediate (15-22) spoke numbers. At 16 spokes, the bit-reversed method produces an evenly-spaced sampling pattern. This is reflected by the lower error in both the Fourier and CS images near 16 projections for the bit-reversed method. Golden-angle sampling performs better at intermediate spoke numbers (19-22) where bit-reversed sampling has an uneven angular spacing. Figure 1b shows a comparison of images reconstructed with 21 projections. It can be seen that for this number of spokes, golden-angle sampling results in a smoother image having the least amount of residual aliasing artifacts when used with compressed sensing reconstruction, compared to the other sampling methods. Random radial sampling consistently performed the worst over the entire range of temporal resolutions studied.

Discussion & Conclusions

In general, CS reconstruction reduces the degree of error caused by streak artifacts in the images, compared to Fourier reconstruction. CS reconstruction also preserves most of the structure (i.e. relative differences between sampling schemes) of Fourier-reconstructed curves when a small amount of noise is added. Bit-reversed sampling produces better Fourier and CS images when the set of spokes is evenly-spaced, while golden-angle sampling converges to better images at other intermediate resolutions.

References

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