Optimization of the b-sampling for bi-exponential analysis of diffusion-weighted imaging

J. L. Zhang[1], E. E. Sigmund[1], H. Chandarana[1], H. Rusinek[1], H. Guo[1], P. Storey[1], Q. Chen[1], and V. S. Lee[1]
[1]Department of Radiology, New York University, New York, NY, United States

Introduction

Diffusion-weighted magnetic resonance imaging (DWI) appears promising for the functional assessment of kidneys (1) and the detection and characterization of renal lesions (2). Recent studies have shown that, in highly vascular tissue, perfusion effect (or "intravoxel incoherent motion (IVIM)" effect) can be identified separately from diffusion through a bi-exponential signal analysis (3). However, bi-exponential analysis is numerically challenging, as resulting parameters have large variability. We explore the precision of the biexponential parameters as a function of diffusion weightings (b values). We calculated optimal b values for DWI of renal lesions, using analytic expressions for noise propagation and Monte Carlo simulations. We hypothesized that, by sampling at the optimal b values, bi-exponential parameters can be estimated with higher precision than at commonly used uniformly distributed b values. We also tested if benign and malignant renal lesions can be accurately differentiated with DWI.

Methods

The bi-exponential model for DWI data is given as

$$S_i = S_0 \cdot [(1 - F_P) \cdot \exp(-b_i \cdot D_T) + F_P \cdot \exp(-b_i \cdot D_P)], \quad [1]$$

where S_i is the signal acquired with diffusion weighting b_i (unit: s/mm^2), S_0 is the signal at b = 0, F_P is the fraction of the perfusion component, D_T is diffusion coefficient, and D_P the pseudo-diffusion coefficient due to perfusion effect. The parameters F_P, D_T and D_P are determined by fitting the acquired signals to Eq. [1]. We also consider a derived parameter, $F_P \times D_P$, which is likely to be more precise than F_P and D_P individually.

The error of each parameter is determined by propagating noisy data during fitting process. Data are assumed to be contaminated with Gaussian noise with assigned variance σ^2. Error propagation ξ for a parameter was defined as relative parameter error normalized to σ. As an example, for parameter F_P is $\xi_{FP} = (\sigma_{FP}/F_P)/(\sigma/S_0)$. For each parameter ξ was derived analytically (4), in terms of the b values (b_i, i = 1 to N) and the expected values of F_P, D_T and D_P. Since expected parameter values for tissues of interest span a known range, our figure of merit is the weighted sum of ξ s integrated over their expected ranges:

$$\xi = \iiint \left(w_{D_T} \xi_{D_T} + w_{F_P} \xi_{F_P} + w_{D_P} \xi_{D_P} \right) dD_T dF_P dD_P \quad [2]$$

Monte Carlo simulation was done to test the benefit of optimized b values in improving differentiation between enhancing and non-enhancing renal lesions: **a)** Based on dynamic contrast enhanced MRI, 18 renal lesions identified in 16 patients were separated into two groups: 10 contrast enhancing (presumed malignant) and 8 non-enhancing (presumed benign). Breath-hold IVIM data were fitted by bi-exponential model (5), yielding expected parameter ranges (Table 1). **b)** The figure of merit ξ (Eq. [2]) was separately minimized for N_b optimal b values, where N_b = 4,5,6,7,8,9,10. Equal weightings *w* in Eq [2] were assumed. An upper bound of b was set to 800 s/mm^2 to achieve sufficient S/N and avoid Rician bias. **c)** S vs. b curves generated for enhancing and non-enhancing lesions based on average values in Table 1 were generated for both optimal b values and b values distributed uniformly between 0 and 800 s/mm^2. Data contaminated with Gaussian noise (σ =0.01 S_0) were repeatedly fitted by bi-exponential model using 1000 independent Monte Carlo trials. **d)** For each parameter, its *effect size* for separating enhancing and non-enhancing lesions was computed as the difference of the mean values divided by the pooled standard deviation.

Table 1. Biexponential parameter values from renal lesions

	Enhancing lesions (10)	Non-enhancing lesions (8)	Ranges for b optimization
F_P (%)	24.8±11.9	8.1±11.6	[5, 30]
D_T (10^{-3} mm^2/sec)	1.46±0.4	2.18±0.7	[1, 3]
D_P (10^{-3} mm^2/sec)	11.0±2.4	12.3±10.9	[10,15]

Table 2. Optimized b values, and their averaged error propagation (ξ). Averaged error propagations of uniformly distributed b values are shown for comparison.

N_b	B_1	b_2	b_3	b_4	Averaged ξ	Averaged ξ of uniform b
4	**0**	**51**	**259**	**800**	121.6	582.9
5	**0**	**56**	2×**258**	**800**	109.4	258.5
6	**0**	**58**	2×**278**	2×**800**	96.6	174.6
7	**0**	2×**55**	2×**262**	2×**800**	88.1	138.8
8	**0**	2×**59**	3×**261**	2×**800**	82.5	119.7
9	**0**	2×**59**	3×**271**	3×**800**	77.0	108.0
10	**0**	3×**59**	3×**263**	3×**800**	73.5	99.7

Results

The optimal distribution of b values consisted of exactly 4 distinct values, regardless of N_b (Table 2), i.e. the optimal protocol involved multiple acquisitions at the same b value. Optimal ξ (Eq [2]) decreased with increasing N_b, and was smaller than ξ for uniformly distributed b values. However, the ξ difference between optimal and uniform b diminished with N_b.

Simulation results are shown in Fig. 1. For F_P, optimal b values yielded an average increase of 22%±7% in the effect size compared to uniformly distributed b for the schemes considered. D_T differentiated the two types of lesion well for both b schemes. The product of F_P and D_P achieved better differentiation between lesions, than F_P and D_P. For $N_b < 7$, the effect size for $F_P \times D_P$ was on average 31% better when using the optimal b values than for uniform b values.

Discussion and Conclusion

Optimization of b values by the proposed method increased precision of bi-exponential parameters, especially perfusion parameters, when the dual compartment IVIM model is assumed to describe tissue behavior. Optimization also appears to improve our ability to indentify malignant (enhancing) lesions. The optimization method is applicable to other organs, provided that typical ranges for the parameters are known a priori. Of particular utility is the ability to select appropriate weightings in Eq [2] to compute b values that are best suited for the parameter of interest.

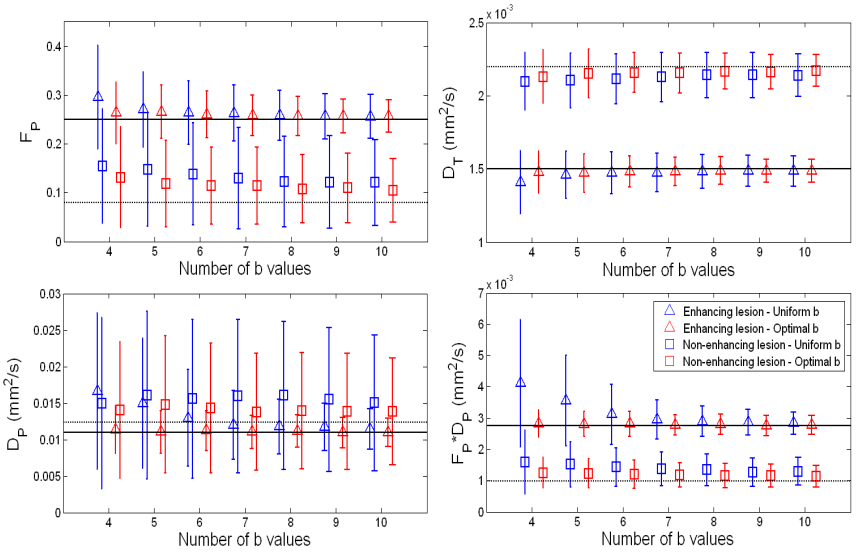

Fig. 1 Bi-exponential parameter estimates of renal lesions in Monte Carlo simulation.

Reference (1) Muller et al. Radiology 1994; 193:711-5. (2) Zhang et al. Radiology 2008; 247: 458-464. (3) Le Bihan et al. Radiology 1988: 168(2): 497-505. (4) Zhang et al. IEEE Trans Biomed Eng 2006; 53(6):1209-14. (5) Chandarana et al. SCBT-MR. Miami, USA, 2009.