A Generalized Diffusivity for Tuning Diffusion-Weighted Imaging Contrast

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Introduction: Parameters derived using diffusion imaging have been widely used for quantitative characterization of tissues' microstructure. One of the main measures derived from diffusion data, the mean diffusivity, ($\overline{D}(t)$), has been found to be a sensitive marker of pathological changes in tissues. $\overline{D}(t)$ is defined as an average of a quadratic function in the diffusion paths lengths of the water protons within a region of interest ($\overline{D}(t) = \langle \delta\mathbf{r}(t) \cdot \delta\mathbf{r}(t) \rangle / 6t$), thus being heavily weighted toward longer diffusion paths. In this abstract, we extend the mean diffusivity concept to introduce a new metric, the generalized diffusivity, which is able to differentially weight diffusion paths of different length scale through an additional scalar parameter, α and present *in vivo* brain imaging results.

Theory: *We define the generalized diffusivity as the regional average of a power function of the water protons' displacement:*

$$D_\alpha^*(t) \equiv \frac{1}{4t}\left\{\Gamma\left(\frac{3}{2}\right)/\Gamma\left(\frac{\alpha+3}{2}\right)\cdot\left\langle\left[\delta\mathbf{r}(t)\cdot\delta\mathbf{r}(t)\right]^{\alpha/2}\right\rangle\right\}^{2/\alpha} \qquad (1)$$

The parameter α may be any real number so that α>-3. For α=2, $D_\alpha^*(t)$ reduces to the conventional mean diffusivity, $\overline{D}(t)$, and for α = −2, $D_\alpha^*(t) = 1/R_D$ where R_D is the diffusional restrictivity (1). For α>2, $D_\alpha^*(t)$ will depend more strongly on the longer diffusion paths than $\overline{D}(t)$, while for α<2, $D_\alpha^*(t)$ will depend more strongly on the shorter diffusion paths than $\overline{D}(t)$. The normalization in Eq. (1) has been chosen so that $D_\alpha^*(t) = \overline{D}(t)$ when the diffusion is isotropic and Gaussian. It can be shown that the $D_\alpha^*(t)$ can be approximated by an integral over the unit sphere of a function of the diffusivity and diffusional kurtosis, $D(\mathbf{u})$ and $K(\mathbf{u})$:

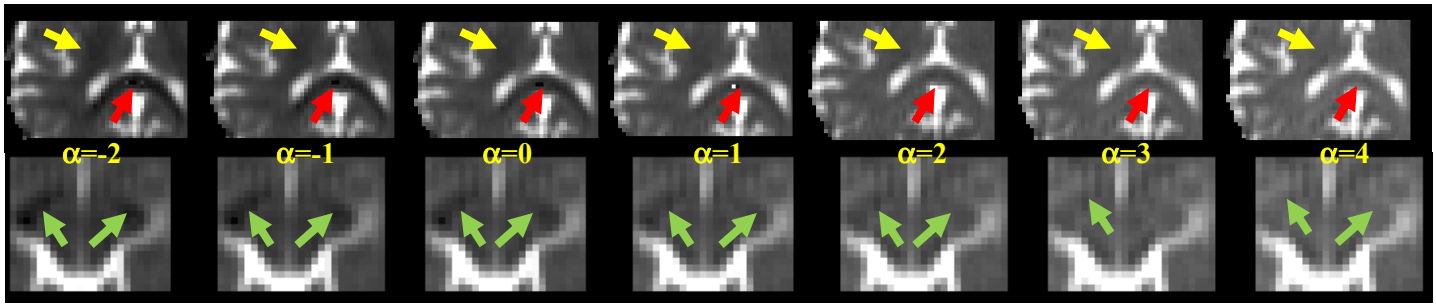
$$D_\alpha^*(t) \approx \left\{(1/4\pi)\cdot\int d^3\hat{u}\,\delta\left(|\hat{\mathbf{u}}|-1\right)\left[D(\hat{\mathbf{u}})\right]^{\alpha/2}\left[1+1/24\cdot\alpha(\alpha-2)K(\hat{\mathbf{u}})\right]\right\}^{2/\alpha}, \quad \alpha \neq 0 \qquad (2)$$

$$D_0^*(t) \approx \exp\left\{(1/4\pi)\cdot\int d^3\hat{u}\,\delta\left(|\hat{\mathbf{u}}|-1\right)\left[\ln\left(D(\hat{\mathbf{u}})\right)-(1/6)\cdot K(\hat{\mathbf{u}})\right]\right\}, \quad \alpha = 0$$

where the diffusivity and diffusional kurtosis for a spatial direction, **u**, are easily obtained from the diffusion and kurtosis tensors (2).

Methods: Imaging experiments were conducted on a 3T Trio MR system for four healthy volunteers. Diffusion weighted (DW) images were acquired for 30 gradient directions and six *b* values (from 0 to 2500 s/mm^2) using a EPI. The in-plane resolution was 2 mm × 2 mm and the slice thickness was 4 mm. The diffusion and kurtosis tensors were calculated similar to (3). The generalized diffusivity values were calculated at each voxel using Eq. (2).

Results: Fig. 1 shows generalized diffusivity maps for different α values. The maps are enlarged in order to facilitate a more detailed visualization of some of the structures. Note that the rate of variation with α depends on the type of tissue with the white/gray matter contrast decreasing as α increases. Regions such as corpus callosum (red arrows), cerebral peduncles (green arrows) and cingulum bundles (not shown) appear to show the largest variation in contrast with varying α. A smaller variation with decreasing α is observed for other white matter structures (e.g., internal capsule, yellow arrows).



Discussion: Sensitive metrics for describing microstructural organization are essential for interpreting diffusivity measurements in both healthy and pathological tissues. In particular, metrics able to describe shorter diffusion paths might be more sensitive to tissues' microstructure and are expected to be less affected than the mean diffusivity by the CSF (which is characterized by longer diffusion paths) in regions with partial volume averaging. Through varying the α parameter, $D_\alpha^*(t)$ offers a new contrast mechanism that may be used to gain increased sensitivity to different types of pathology or to gain insight regarding the source of diffusion differences that may occur between different tissue types or populations.

References: 1. Lazar M et al, ISMRM 2007. 2. Jensen JH et al. MRM 2005;53:1432. 3 Lu H, Jensen JH, et al. NMR Biomed 2006; 19:236.