Bias free estimates of the diffusional kurtosis in two minutes: Avoid solving the kurtosis tensor

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Introduction

Diffusion kurtosis tensor imaging provides additional information compared with diffusion tensor imaging [1,2]. Solving the kurtosis tensor, however, requires measurements in at least 15 diffusion-encoding directions as well as measurements with at least three or more *b*-values in each of these directions [2]. This leads to scanning times in the order of 6-10 minutes, which is often too long for incorporation in a routine clinical protocol. We suggest a method to estimate a bias free kurtosis value from only six diffusion encoding directions, thus enabling assessment of the kurtosis in less than two minutes of scan time.

Theory

The kurtosis parameter is related to the echo signal (S) in the i:th diffusion encoding direction according to

$$S_i(b) = S_0 \cdot \exp(-b \cdot ADC_i + b^2 ADK_i \cdot ADC_i^2/6), \quad (1)$$

where S_0 is the signal magnitude at b = 0, b reflects the degree of diffusion encoding, ADC is the apparent diffusion coefficient and ADK is the apparent diffusion kurtosis. The relationship between these parameters, measured in different directions, is given by an invariant diffusion kurtosis (DK) tensor of rank four, denoted W, and consisting of 15 unique elements [3]. The kurtosis in each direction x_i is obtained from the tensor, according to

$$ADK_i \cdot ADC_i^2 = MD^2 \cdot Wx_i^4, \quad (2)$$

where MD is the mean diffusion and Wx_i^4 is given by $W_{klmn} \cdot x_k x_l x_m x_n$ summarized over k, l, m and n = 1, 2, 3 [4]. The mean kurtosis (MK) is obtained as the average ADK_i over all measured directions [1]. Averaging Wx_i^4 over all measured directions yields MW, i.e. the tensor average.

As an alternative to solving the kurtosis tensor, it has been suggested that the kurtosis (DK_G) can be approximated by fitting $S_G(b) = S_0 \cdot \exp(-b \cdot MD + b^2 DK_G \cdot MD^2/6)$ to the measured signal S_G, geometrically averaged over all measured directions [5]. Note that Eq. 2 implies that $DK_G = MW$, since averaging $ADK_i \cdot ADC_i^2$ over the measured directions i can be approximated by $MD^2 \cdot MW$.

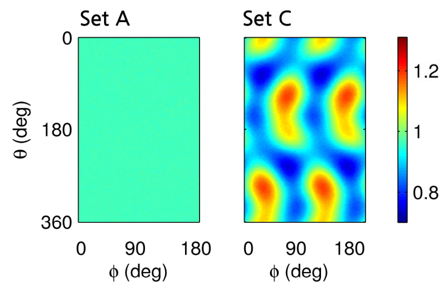

Fig. 1. DK_G estimated while rotating the employed gradients around the *z* and the *x*-axes. Left: Using an optimized gradient scheme with six encoding directions (Set A), DK_G was estimated to be 0.96 without any noticeable rotation-dependent bias. Right: The non-optimized set (Set B) results in a bias in DK_G of up to approximately ±20%.

Method

A reduced set of six diffusion encoding directions, denoted A, was created by the method of electrostatic repulsion [6]. These six directions were duplicated into three subsets, which again all were subjected to the method of electrostatic repulsion, but this time fixing the mutual directions in each subset. This produced a combined set of 18 directions, denoted set B. For comparison, a non-optimal set C was also included in the investigations, consisting of six diffusion encoding directions according to (x,y,0), (-x,y,0), (0,y,z), (0,-y,z), (x,0,z) and (-x,0,z).

Using the directions in sets A, B and C, synthetic diffusion measurements based on an example kurtosis tensor were generated. From these data, DK_G was calculated on the basis of both sets A and C. MK and MW were calculated from the fitted kurtosis tensor for set B. The calculations were performed for different rotations of the gradient coordinate system, in order to study potential directionally related bias in the estimated DK_G.

Diffusion weighted images were acquired at a 3T Philips Achieva system, using a single shot EPI pulse sequence with TE/TR = 77/4792 ms/ms, b = 0, 1.0, 1.5, 2.0 and 3.0 ms/µm², using diffusion encoding directions according to set B. Maps of DK_G were calculated from the first six directions in B, i.e. subset A. Maps of MK and MW were calculated from the kurtosis tensor, based on dataset B.

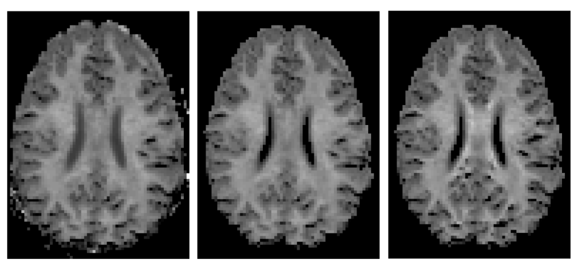

Fig. 2 (above). Left: DK_G estimated from the reduced set of six optimized diffusion encoding directions (set A). Middle and right: MW and MK, respectively, obtained from the full kurtosis tensor (set B). The image contrasts in DK_G and MW are highly similar, while MK and MW differ as can be seen, for example, in the corpus callosum region.

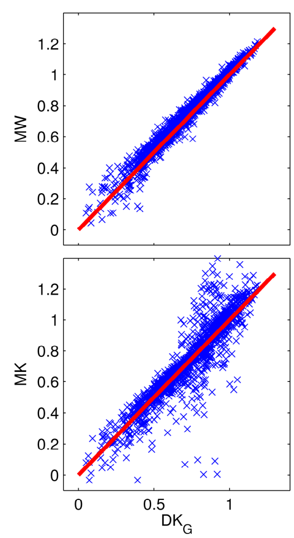

Fig. 3 (right). Top: Scatter plot showing MW versus DK_G. The obtained values show a high conformity for the higher kurtosis values, but MW was, in general, slightly higher than DK_G. Bottom: A scatter plot showing MK versus DK_G. The tensor parameters MW and MK represent, as expected, slightly different information and this is reflected by the scatter plot.

Results

Using the optimized measurement directions in set A, the obtained value of DK_G was independent of the coordinate system rotation. For the non-optimal dataset C, however, the direction of the coordinate system influenced the obtained value of DK_G and induced a maximal bias of approximately 20% (Fig. 1). Solving the kurtosis tensor showed that MW obtained from set B was equal to DK_G obtained from set A. In Figs. 2 and 3, the parametric maps and scatter plots indicate that DK_G (set A) and MW are similar. However, MK did not equal DK_G as expected.

Discussion and conclusion

We have shown that the kurtosis value (DK_G) based on a geometrically averaged signal value corresponds to the average value of the kurtosis tensor (MW), shown in Figs. 2 and 3. When estimating DK_G, it is important that the diffusion encoding directions are optimized using the method of electrostatic repulsion, otherwise a substantial bias in the DK_G estimate may occur (Fig. 1).

In conclusion, a bias-free kurtosis value may be obtained in a total scan time of two minutes, making the kurtosis parameter assessable in a clinical setting.

References

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