A temperature distribution model for gradient coils

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Introduction: High gradient fields demand high coil currents and this leads to high local Joule heating in the gradient coils due to their resistive material properties. Excessive gradient heating is a considerable concern as it can lead to heat generation in nearby devices, such as shimming trays or radiofrequency coils, which can in turn lead to image distortion and signal loss. Furthermore, intense local heating can cause failures in localised parts of the gradient coil. Typically, a cooling mechanism is integrated into the system in the form of a piping network, which removes heat via forced water cooling (*eg* [1]). Very few models exist in the literature for describing the temperature within the gradient system. Chu and Rutt [2] model the average temperature of a gradient coil over time and Leggett *et al.* [3] model the average temperature for a multi-layer gradient design. An analytic model is proposed here for the spatial temperature distribution over the surface of a gradient coil. That is, local temperature variations can be predicted including the location of gradient hot spots, apparently for the first time. The model allows many coil parameters to be varied, including coil geometry and thermal properties, and temperature distributions can be calculated under convective air, forced air, or forced water cooling.

Method: The gradient coil is represented by a cylindrical copper sheet of radius r_c, length $2L$ and thickness w. This copper cylinder is assumed to be embedded in an epoxy former that extends radially outwards to a radius r_o and inwards to r_i. A heat equation is constructed by considering Ohmic heating as a result of current density $\boldsymbol{j}$ (A/m), heat conduction throughout the copper layer, radial conduction through the former, and radial convection and radiation to the environment (see While *et al.* [4]):

$$\rho_d c_h \frac{\partial T^*}{\partial t} = k_c \nabla^2 T^* + \frac{\rho_r}{w^2}\boldsymbol{j}\cdot\boldsymbol{j} - \frac{h_t}{w}T^* \qquad (1)$$

$$h_t = \left[\frac{\Delta r_i}{k_f} + \frac{r_c}{r_i h_i}\right]^{-1} + \left[\frac{\Delta r_o}{k_f} + \frac{r_c}{r_o h_o}\right]^{-1} + h_r\left(\varepsilon_i \frac{r_i}{r_c} + \varepsilon_o \frac{r_o}{r_c}\right) \qquad (2)$$

Here T^* is the temperature difference between the coil and the environment, ρ_d, c_h, k_c and ρ_r are the density, specific heat, thermal conductivity and resistivity of the copper, h_t is the total heat transfer coefficient, which represents the radial cooling in the system (see also [2]), k_f is the thermal conductivity of the former, h_i and h_o are heat transfer coefficients for the cooling mechanism at the inner and outer surfaces of the former, h_r is the approximate radiative heat transfer coefficient and ε_i and ε_o are the emissivities of the inner and outer surfaces, respectively (see [4]). To investigate the spatial distribution of the temperature it is sufficient to solve for the steady-state solution of Eq. (1) provided this is reached within a sufficiently small time frame. Setting the right-hand side of Eq. (1) to zero and rearranging, we obtain a 2D screened Poisson equation. This is solved in two ways: firstly using Green's functions, and secondly using Fourier series, provided care is taken to solve both homogeneous and inhomogeneous forms of the equation (see [4]). In addition, an approximate solution for the time-dependence of the maximum coil temperature is obtained by linearising the first term on the right hand side of Eq. (1). This provides a further check for the steady-state temperature maximum and allows rise-times to be investigated to check that thermal equilibrium is reached within a typical scanning scenario. A simpler method for the average temperature was used by Chu and Rutt [2] and was verified experimentally.

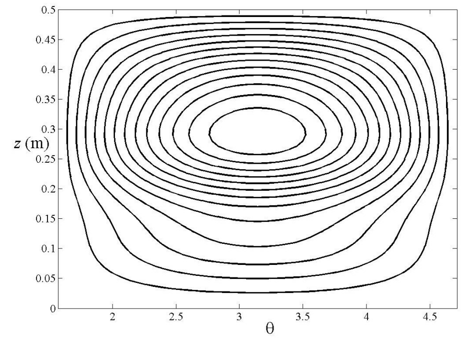

Fig. 1: Coil windings for minimum power *x*-gradient.

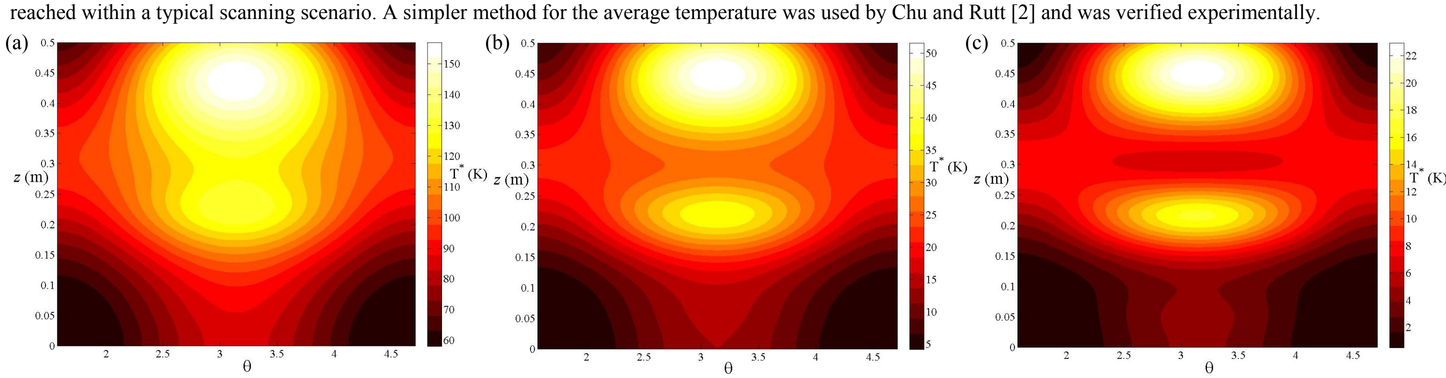

Fig. 2: Spatial temperature distributions for minimum power *x*-gradient coil example with (a) convective air, (b) forced air, and (c) forced water cooling.

Results: The present model allows a great number of coil properties to be varied to study their impact on the spatial temperature distribution and rise-time. A 50 mT/m *x*-gradient coil example is considered, of radius $r_c = 0.25$ m and length $2L = 1$ m. Coil windings obtained using a minimum power method similar to Forbes and Crozier [5] are shown in Fig. 1. The following examples were calculated using $w = 0.002$ m, $\Delta r_i = \Delta r_o = 0.002$ m, $k_f = 0.6$ W/m/K, $\varepsilon_i = \varepsilon_o = 0.9$ and $h_r = 7.8$ W/m^2/K. Fig. 2 displays the temperature distributions for the gradient coil depicted in Fig. 1 under (a) convective air cooling ($h_i = h_o = 10$ W/m^2/K), (b) forced air cooling ($h_i = h_o = 100$ W/m^2/K), and (c) forced water cooling ($h_i = h_o = 1000$ W/m^2/K). Hot spots are found to occur where the coil windings are closely spaced, as expected. The effect of cooling is clear, with hot spot temperature (above ambient) decreasing from (a) 156.9 K to (b) 51.6 K to (c) 22.9 K. In addition, the distribution has more clearly defined hot spots with greater cooling as a result of the increased rate of radial heat transfer to the environment relative to the dissipative conduction of heat throughout the copper layer. The rise-times necessary to reach 95% of the steady-state distribution for each type of cooling are (a) 227 s, (b) 71 s and (c) 31 s, respectively. Heavily insulated cases that more closely approximate the regime of individual coil wires embedded in epoxy resin within the former can be considered by altering k_c (see [4]). In summary, the temperature of the gradient coil can be minimised in an engineering sense, by constructing a coil made from a thick copper sheet embedded in a thin former of high thermal conductivity with forced water cooling. This general finding may have been expected; nevertheless, the model presented offers great utility in predicting the effect of varying a whole suite of coil system parameters, not only on the spatial distribution of the temperature but also the temporal behaviour of the hot spots. The temperature distributions such as those displayed in Fig. 2 also aid in finding the optimum placement of cooling pipes when constructing gradient coils.

Conclusion: A method has been presented for calculating the spatial distribution of temperature within a gradient coil system and the time-dependence of the hot spot temperature. Numerous coil parameters may be varied in the model, providing great utility prior to manufacture. Subsequent to this work, the temperature distribution has been utilized in a non-linear iterative optimization routine for redesigning gradient coils with reduced hot spot temperatures at no cost to coil performance [4].

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