# Minimising hot spot temperature in gradient coil design

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**Introduction:** In the operation of gradient coils, local Joule heating due to high coil currents is a considerable concern and can lead to image distortion or damage to the coils. Typically, cooling pipes carrying water are included in the gradient system to ensure coil temperature remains below an acceptable level (*eg* [1]). Very few design methods in the literature consider the temperature of gradient coils in their optimisation routines. Poole *et al.* [2] target coil spacing directly by manually manipulating matrix elements in a boundary element method, and Leggett *et al.* [3] develop an average temperature model for a multi-layer gradient design and weight a power constraint to optimise layer position and enhance cooling. An optimisation strategy is proposed here that targets the spatial temperature distribution over the coil cylinder directly, with the aim of obtaining gradient coils with reduced hot spot temperatures. This is a highly non-linear problem as it involves a maximum temperature constraint and this is minimised using a relaxed fixed point iteration routine.

**Method:** A cylindrical $x$-gradient coil is considered of radius $r_c$ = 0.25 m and length $2L$ = 1 m. While *et al.* [4] model the spatial temperature distribution of this coil type by considering a cylindrical copper sheet of finite thickness embedded within an epoxy former. The model includes consideration of Ohmic heating by a current density $\boldsymbol{j}$ (A/m), heat conduction throughout the copper layer, radial conduction through the former, and radial convection and radiation to the environment. Here we consider the total square of the gradient of this temperature distribution as an appropriate constraint for minimising hot spot temperature (see [4]). As this constraint is not quadratic with respect to current density, such linear techniques as Tikhonov regularisation are not available, and an iterative optimisation scheme must be devised. This requires an initial starting guess, sufficiently close to the optimum solution, and a minimum power coil as described in Forbes and Crozier [5] is used for this purpose and for subsequent comparison. A functional involving field error, coil power and the maximum temperature constraint is minimised at every iteration. This is accomplished using a two-step numerical scheme (see [4]):

1) Solve: $(A + \lambda_P P)\, \boldsymbol{X_i} = \boldsymbol{T} + \lambda_Q \boldsymbol{Q}(\boldsymbol{x_i})$
2) Update: $\boldsymbol{x_{i+1}} = \omega\, \boldsymbol{x_i} + (1 - \omega)\, \boldsymbol{X_i}$.

Eq. (1) is a linear matrix equation whose solution minimises the functional described above. Matrix $A$ and vector $\boldsymbol{T}$ contain target field conditions, matrix $P$ contains minimum power conditions with regularising parameter $\lambda_P$, and vector $\boldsymbol{Q}$ contains the minimum hot spot temperature conditions with weight $\lambda_Q$ and is evaluated using the current density coefficients from the previous iteration, which are contained in vector $\boldsymbol{x_i}$. Iteration is necessary due to the non-linear constraint and Eq (2) is in the form of a relaxed fixed point iteration routine, with relaxation parameter $\omega$, and vector $\boldsymbol{x_{i+1}}$ contains updated current density coefficients. Care must be taken to choose appropriate $\lambda_Q$ and $\omega$ values to ensure convergence (see [4]).

| Table 1 | $x_0$ | $x_1$ | $x_2$ | ….. | $x_8$ |
|---|---|---|---|---|---|
| max($T^*$) (K) | 51.6 | 44.7 | 41.3 | ….. | 39.4 |
| norm($x_{i+1}$ - $x_i$) | - | $3.1\times10^2$ | $7.5\times10^1$ | ….. | $5.2\times10^0$ |

**Table 1:** Hot spot temperature for current density $\boldsymbol{x_i}$ ($i$=0:8) at each iteration of Eqs. (1) and (2).

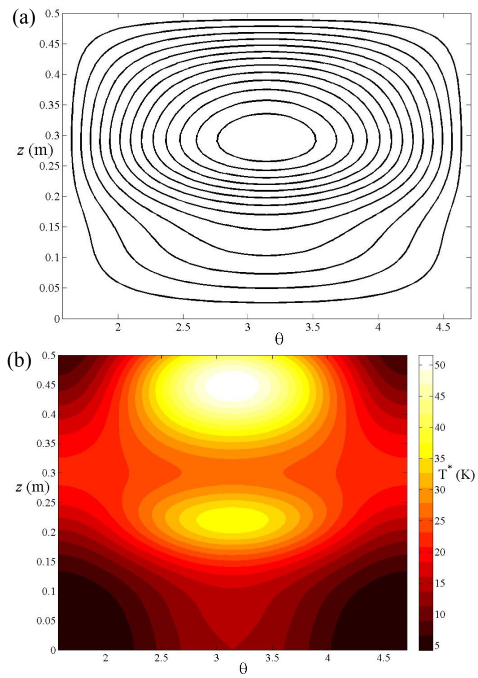


**Fig. 1:** (a) Coil windings for min. power $x$-gradient. (b) Spatial temperature distribution for 1(a).

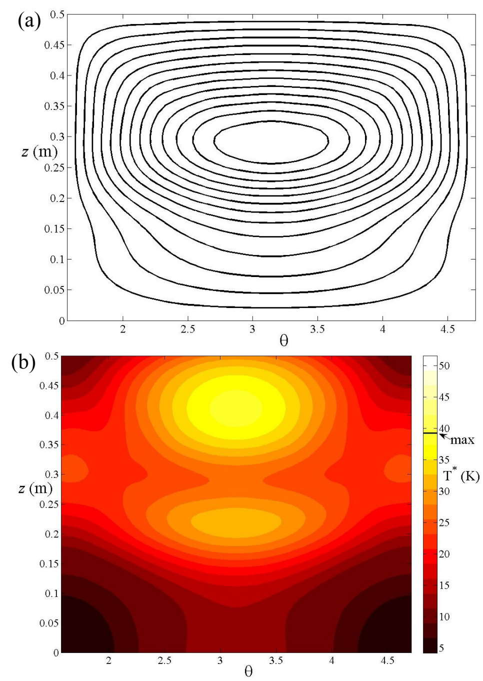


**Fig. 2:** (a) Coil windings for min. hot spot coil. (b) Spatial temperature distribution for 2(a).

**Results:** Fig. 1(a) displays the coil windings in one quadrant of a standard minimum power $x$-gradient coil, with field error $\delta^{1/2}$ = 0.69% (0.3 m DSV), efficiency $\eta$ = 106 μT/A/m and inductance L = 216 μH, such that the coil performance figure of merit $\eta^2/L$ = 52.5 $\mu T/A/m^4$. The corresponding temperature distribution under forced air cooling, found using the model of [4], is displayed in Fig. 1(b), and the hotspot temperature is found to be max($T^*$) = 51.6 K (above ambient). Using the minimum power result as the initial guess $\boldsymbol{x_0}$ in the iterative optimization scheme of Eqs. (1)-(2), with $\lambda_P = -5\times10^{-15}$ and $\omega$ = 0.8, we observe a drop in hot spot temperature over 8 iterations as shown in Table 1. That is, we observe a 13.3% drop in the first iteration alone and in total a 23.6% drop to give max($T^*$) = 39.4 K. The column norm($\boldsymbol{x_{i+1}} - \boldsymbol{x_i}$) of Table 1 confirms the convergence of the current density solution. Fig. 2(a) shows the coil windings in one quadrant of the coil after 8 iterations. In comparison to Fig. 1(a) we note a spreading of the windings in the denser regions of the coil and a squaring off in other regions to accommodate this redistribution. The corresponding temperature distribution is shown in Fig. 2(b), which displays a much greater spreading of the hot spots and the lower maximum temperature. The improvement in hot spot temperature does come at the cost of an increase in field error $\delta^{1/2}$ = 0.86% and inductance L = 220 μH, but an improved efficiency $\eta$ = 110 μT/A/m and hence higher $\eta^2/L$ = 55.0 $\mu T/A/m^4$. However, a more appropriate comparison is to redesign a minimum power coil with the same increased field error. This is found to have identical coil performance $\eta^2/L$ to the minimum hot spot temperature coil but a much greater hot spot temperature of 46.8 K. That is, for equivalent field error and coil performance, the minimum hot spot method results in a maximum temperature value that is 15.8% lower than that obtained by the minimum power method. Similar results are found consistently for other coil types, for instance with different types of cooling, geometry or thermal properties, with some examples showing over 20% improvement in hot spot temperature (see [4]). Note that the optimisation method could be adapted in a straightforward manner to consider other non-linear constraints.

**Conclusion:** A model has been presented for designing gradient coils with improved temperature distributions and lower hot spot temperatures, when compared to standard minimum power gradient coils, at no cost to coil performance. The model provides great utility in examining the temperature profile of gradient coils prior to construction and offering a means of adjusting the locations of coil windings to reduce hot spot temperature considerably. In addition, non-linear constraints other than maximum temperature may be included easily in the optimisation routine and the method is adaptable to other coil geometries.

**References:** [1] B. Chronik *et al.*, *Magn. Reson. Med.*, vol. 44(6), pp. 955-963, 2000.
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