Eigenmode Analysis of Eddy Currents and Eigenmode Coil Design

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Introduction Eddy currents have long been the bane of MRI. Particularly since the mid 1980s when imaging sequences became more rapid [1] and superconducting magnets became routinely used. When gradient coils are switched rapidly, eddy currents are induced in the cryostat vessel and other surrounding conducting structures. Two principal solutions have been adopted into standard practice for dealing with the spatiotemporal eddy current field problem: "pre-emphasis" aims to temporally modify the gradient pulse such that the primary gradient field and it's secondary field combine to produce the desired pulse shape [2]; and "active shielding" or "active screening" in which an additional coil is interposed between primary gradient coil and cryostat to reduce the fringe field to almost zero [3]. Pre-emphasis only works well for one point in space since the eddy current field is not identical to the primary field and changes in time. Active shielding is never perfect with some residual field present in the ROI with complex spatiotemporal behaviour. Both approaches are often used in combination in most MRI scanners. In this work we investigated the nature of the eddy currents induced in surfaces and their relation to the currents in the gradient coil by simulated eigenmode analysis [4]. Additionally, we study a simple solution to improving pre-emphasis by reducing the problem to a purely temporal one.

Methods Numerical analysis of a passively-shielded gradient coil arrangement was simulated with a boundary element method [4]. Experimental validation of the simulations was attempted by construction of a model Z-gradient coil; field measurements were made on axis using a magnetic tunnel junction (MTJ) magnetoresistive sensor [5] under trapezoidal current pulsing [6]. The equation that links the source currents (defined by the vector $\mathbf{s}_s$) with the induced eddy currents ($\mathbf{s}_i$) is given by Eq. (1) and the solution to the initial value problem is Eq. (3) using the matrices from the generalised eigenvalue problem (2).

$$\mathbf{M}_{is}\frac{d\mathbf{s}_s}{dt}+\mathbf{M}_{ii}\frac{d\mathbf{s}_i}{dt}+\mathbf{R}_{ii}\mathbf{s}_i=0 \quad (1), \qquad \mathbf{R}_{ii}\mathbf{U}=\mathbf{M}_{ii}\mathbf{U}\mathbf{\Lambda} \quad (2), \qquad \mathbf{s}_i(t)=\mathbf{U}e^{-\mathbf{\Lambda}t}\mathbf{U}^{-1}\mathbf{s}_i(0)-\mathbf{U}\int_0^t e^{-\mathbf{\Lambda}(t-\tau)}\mathbf{U}^{-1}\mathbf{M}_{ii}{}^{-1}\mathbf{M}_{is}\frac{d\mathbf{s}_s(\tau)}{d\tau}d\tau \quad (3).$$

$\mathbf{M}_{is}$ is the mutual inductance between gradient coil and passive shield surfaces and $\mathbf{M}_{ii}$ and $\mathbf{R}_{ii}$ are the self-inductance and resistance of the passive shield, respectively. $\mathbf{\Lambda}$ is a matrix whose diagonal elements contain the reciprocal time constants (eigenvalues) and $\mathbf{U}$ contains the eigenmodes of the eddy currents. This analysis is interesting because it elucidates the spatiotemporal behaviour of the eddy currents in terms of non-interacting, purely exponentially-decaying eddy current modes. We analyse these modes to observe their form and calculate the magnetic field that they generate. **From these analyses it is evident that the complex spatiotemporal eddy current field variation may be reduced to a purely temporal variation by ensuring that the gradient coil excites purely one eddy current mode and itself produces a field which matches that of the excited mode.** If this is possible, then pre-emphasis would work exactly over the whole region of interest with a single time constant filter since the spatial form of the eddy current field stays the exactly same. We designed "eigenmode coils" to do this and analysed their behaviour by simulation. Suppression of all but one mode was achieved using Lagrange multipliers added to the coil design functional [7,8].

Results Figure 1 shows the stream-function of a few of the low energy (those with long time constant decays) eddy current modes in a finite length cylinder with their magnetic field plotted over the ROI surface in Fig.2. Figure 3 show the wire paths of a **single layer** coil designed to purely excite one mode (#8) and to generate the magnetic field of that mode. The field from mode #8 has a maximum error of 14% with respect to a linear gradient field. Figure 4 shows the temporal behaviour of the maximum of the residual field after a 1 Amp "switch off" gradient pulse with pre-emphasis.

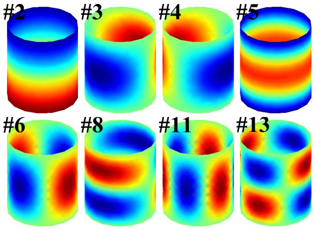

Figure 1. Some stream-functions of the eddy current modes of a finite-length cylinder.

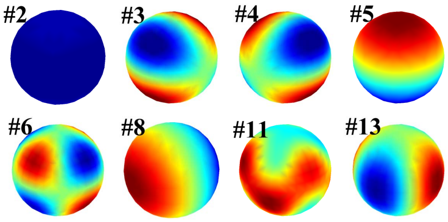

Figure 2. Magnetic fields produced by the eddy current modes shown in Fig. 1.

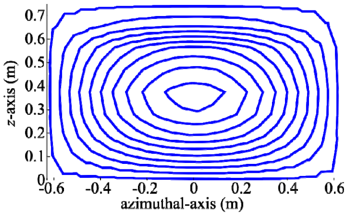

Figure 3. Wire-paths of one quadrant of a #8 mode coil.

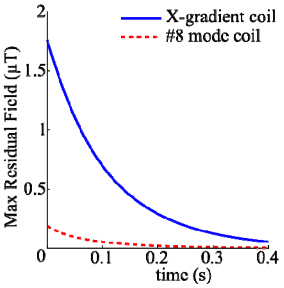

Figure 4. Residual field of unshielded X-gradient and #8 mode coils

Discussion and Conclusions Equations (1) to (3) and Figs. 1 and 2 elucidate some interesting physical properties of the modal behaviour of eddy currents. The spatial form of the modes appears to closely resemble that of cylindrical and spherical harmonics. Since the modes are orthogonal, the fields that are produced by each mode are also orthogonal and represent a basis set similar to spherical or cylindrical harmonics for finite-length cylindrical systems. Modes exist with degenerate eigenvalues (e.g #3 and #4 in Fig. 1) corresponding to modes that are rotations of each other. Eigenmode fields are less linear than gradient coils, but in combination with distortion correcting reconstruction techniques [9] may provide a more predictable Fourier encoding basis set for fast imaging or diffusion studies. The mode coil is strikingly similar to a conventional X-gradient coil with somewhat modified wire path shape at the ends of the coil. It is thought that these wires ensure that the primary and secondary magnetic fields match well. It appears possible to improve the spatial performance of pulse pre-emphasis with single layer coils which require less space within MRI scanner, but gradient amplifiers with large current overheads would be required for the large pre-emphasis currents. Because the modes are dictated by the eddy current surface it may be possible to somewhat improve the linearity of their fields by modification of this surface [6]. Although the results are simulated we have begun to validate the simulations with experimental measurement of the eddy currents [6] using a MTJ sensor [5]. In real MRI scanners multiple layers exist which will cause multiple exponential decays. These surfaces are of similar geometry and therefore the spatial form of their eddy current modes will also be similar and not present much more of a challenge to pre-emphasis with the eigenmode coils. It is possible to use two surfaces for the mode coil to also incorporate active shielding. Since the eddy current modes are non-interacting, the mode coil concept should be extendable to more complex modes allowing them to be used in combination without mutual inductive interaction which will have advantages in a number of applications. We employed a triangular BEM to study these phenomena which allows simulation of arbitrary shaped surfaces.

References [1] Mansfield, P. *J Phys. C*, (1977), **10**, L55-L58 [2] Glover, G. H. & Pelc, N. J. *US Patent* 4,698,591, (1987) [3] Mansfield, P. & Chapman, B. *J Phys E*, (1986), **19**, 540-545 [4] Peeren, G. N. *J Comp Phys*, (2003), **191**, 305-321 [5] STJ-201, Micro Magnetics, Inc. Fall River, MA [6] Sanchez Lopez, H., Poole, M. & Crozier, S. *submitted to ISMRM 2010* [7] Lemdiasov, R. A. & Ludwig, R. A. *Concepts Magn Reson B*, (2005), **26B**, 67-80 [8] Poole, M. & Bowtell, R. *Concepts Magn Reson B*, (2007), **31B**, 162-175 [9] Lai, C.-M. *J Phys E*, (1983), **16**, 34-38.

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