Compressed Sensing with Transform Domain Dependencies for Coronary MRI

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INTRODUCTION: Long data acquisition time of coronary MRI has been one of the main limitations of this technique. Parallel imaging has been used to accelerate image acquisition in coronary MRI but with limited success due to noise amplification during image reconstruction. Compressed sensing (CS) has been recently utilized to accelerate image acquisition in MRI [1,2]. However, its use in cardiac MR has been limited due to blurring that results from imposed CS constraints. In this study, we sought to develop an improved CS reconstruction method that uses the dependencies of the transform domain coefficients to reduce the observed image blurring and artifacts in coronary MRI.

THEORY: A Gaussian scale mixture (GSM) model is used to develop a Bayes least squares (BLS) soft-thresholding function [3]. This model captures the dependencies and sparsity of the wavelet coefficients. The acquired data in the j^{th} coil is given by $\mathbf{y}_j = F_\Omega(C_j \times \mathbf{u}) + \mathbf{n}_j$, where $\mathbf{u}$ is the image, C_j is the coil sensitivity, F_Ω is the partial unitary Fourier operator, and $\mathbf{n}_j$ is measurement noise. CS reconstruction minimizes an objective function of the form, $\|\mathbf{y} - F_\Omega(\mathbf{d})\|^2 + \tau\,\Phi(\mathbf{d})$, which combines a fidelity measure of image consistency and a weighted regularizer (typically l_p norm, $p \leq 1$) that captures the sparsity of the image in a transform domain (e.g. wavelet). This regularization can be viewed in Bayesian setting with $p(\mathbf{u}) \sim \exp(-\tau\Phi(\mathbf{u}))$ as the probability density function of the transform coefficients. Regularizers based on l_p norms correspond to independent and identically distributed wavelet domain coefficients, and give rise to thresholding functions that act on each wavelet coefficient separately [4]. However, for MR images, there is correlation between the wavelet coefficients of a given neighborhood that includes surrounding coefficients from the same subband, as well as neighboring coefficients from nearby scales (Fig. 1). Both this correlation and the sparseness of the wavelet transform can be captured using a Gaussian scale mixture (GSM) model [3]. A random vector $\mathbf{x}$ is a GSM if it can be expressed as $\mathbf{x} = z^{1/2}\boldsymbol{\theta}$, where $\boldsymbol{\theta} \sim N(\mathbf{0},\mathbf{C}_\theta)$ is a zero-mean Gaussian vector and z is a multiplier random variable. In our model, we let z be distributed with Jeffrey's non-informative prior [3], $p_z(z) \sim 1/z$. Let x_C be the wavelet coefficient, $\mathbf{h}$ be the wavelet coefficients of the current image estimate we want to threshold, and $\mathbf{w}$ be the wavelet coefficients of the noise in this neighborhood. We note that $\mathbf{w}$ depends only on the measurement noise $\mathbf{n}$ [5], which is modeled as Gaussian, $\mathbf{w} \sim N(\mathbf{0},\mathbf{C}_\mathbf{w})$. BLS estimator of x_C is $x_C^{thr} = \boldsymbol{E}(x_C|\mathbf{h}) = \int_0^\infty \boldsymbol{E}(x_C|\mathbf{h},z)\, p(z|\mathbf{h})\, dz$ [3]. For the GSM model and Gaussian noise, $\boldsymbol{E}(\mathbf{x}|\mathbf{h},z) = z\mathbf{C}_\theta(z\mathbf{C}_\theta+\mathbf{C}_\mathbf{w})^{-1}\mathbf{h}$. $p(z|\mathbf{h})$ can be calculated via Bayes' rule from $p(\mathbf{h}|z)$ and $p_z(z)$ which are known. The thresholded value for the central coefficient is calculated via numerical integration using the above formula for x_C^{thr}.

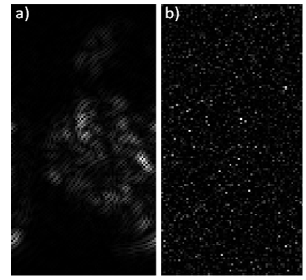

Fig. 1: a) Wavelet coefficients of a 2D slice of a coronary image. b) Random permutation of the same coefficients shown in (a). Both data have equivalent l_p norm, which suggests CS l_p norm regularizers do not take into account the clustering and correlation of information in the transform domain.

METHODS: The proposed reconstruction method was implemented in Matlab. The reconstruction algorithm include the following steps: 1) Initialize $\mathbf{u}^{(0)}$ to be the low-resolution image. 2) At iteration t: (a) map $\mathbf{u}^{(t)}$ to coil j, $\mathbf{u}_j^{(t)} = C_j\mathbf{u}^{(t)}$; (b) calculate $\mathbf{U}_j^{(t)} = F(\mathbf{u}_j^{(t)})$; (c) set the values of $\mathbf{U}_j^{(t)}$ at locations Ω to be $\mathbf{y}_j$; (d) compute the F^{-1} of (c); (e) combine the coil images via rate-1 SENSE; (f) perform BLS-GSM thresholding for each subband of the wavelet coefficients, generating $\mathbf{u}^{(t+1)}$. For each subband, we estimate the neighborhood covariance $\mathbf{C}_\theta$ using the sample covariance matrix from noisy data. The noise covariance $\mathbf{C}_\mathbf{w}$ is computed a-priori based on the measurement noise distribution, which implicitly includes the threshold value. We use neighborhoods including the parent wavelet coefficient and the 3×3 neighborhood surrounding x_C in the same subband. Phantom Evaluation: Fully sampled k-space data (240 phase encode lines) were acquired in a resolution phantom. The relative B_1 coil map was reconstructed from a low-resolution scan (spatial resolution=2×2 mm^2) by zero-padding. A region-of-interest was specified, and coefficients outside this region were penalized in the reconstruction. The k-space data was under-sampled by factors of 2 and 4 by keeping 16 phase-encodes around the center. Coronary MRI: A 3D, free-breathing ECG-triggered SSFP (TE/TR/α=4.3/2.1/90°, spatial resolution=1×1×3 mm^3) sequence with T_2-prep and fat saturation was used for imaging the right coronary artery. The relative B_1 coil map was reconstructed from the fully-sampled data without any further post-processing. The k-space data were under-sampled by factors of 4, 6 and 8 by keeping 10 to 16 phase-encode lines around the center while randomly discarding data in the outer region. Images were reconstructed using both the proposed method and l_1 norm minimization [2] using steerable pyramids as wavelet transforms [3].

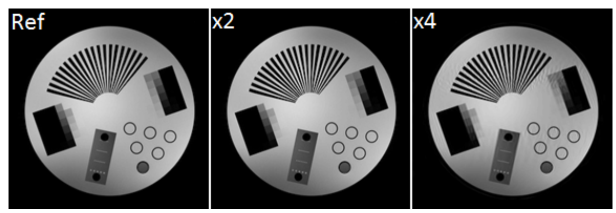

Fig. 2: BLS-GSM CS reconstruction applied to resolution phantom.

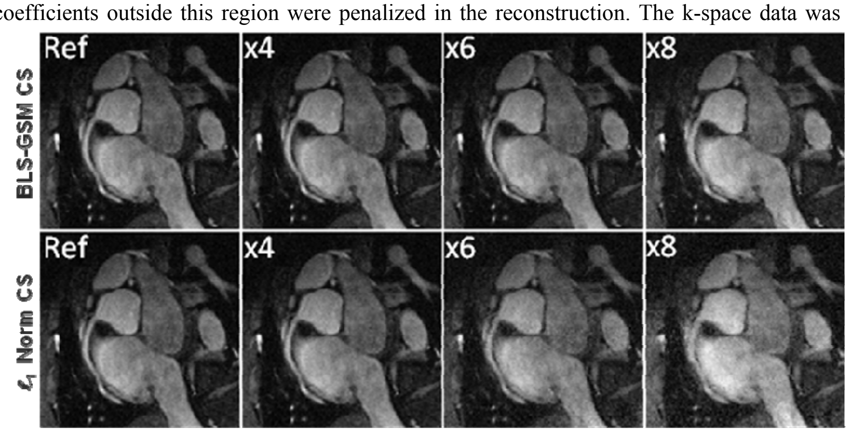

Fig 3: Comparison of BLS-GSM CS and l_1 norm CS for imaging of right coronary artery.

RESULTS: Fig. 2 depicts the results of BLS-GSM method on phantom data. The results of coronary data are shown in Fig. 3, which show visible improvements and reduced coronary blurring at higher acceleration rates compared to l_1 norm.

CONCLUSIONS: We have demonstrated the feasibility of CS coronary MRI with transform domain dependencies with reduced image blurring.

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