Fast MR parameter mapping from highly undersampled data by Direct Reconstruction of Principal Component Coefficient Maps using Compressed Sensing

C. Huang[1], C. Graff[2], A. Bilgin[3], and M. I. Altbach[4]

[1]Mathematics, University of Arizona, Tucson, Arizona, United States, [2]Program in Applied Mathematics, University of Arizona, [3]Biomedical Engineering, University of Arizona, [4]Radiology, University of Arizona

Introduction: There has been an increased interest in quantitative MR parameter mapping techniques which enable direct comparison of tissue-related values between different subjects and scans. However the lengthy acquisition times needed by conventional parameter mapping methods limit their use in the clinic. For estimating T2 values from highly undersampled radial Fast Spin Echo (FSE) datasets, an echo sharing method was proposed in [1]. While this method produces accurate T2 estimates for large objects, the estimation error increases for smaller structures such as small tumors. Recently, model-based techniques that utilize sparsifying penalty functions as suggested by the Compressed Sensing (CS) theory [2] have also been proposed for this task [3,4]. In this work, we introduce a new model-based approach to reconstruct accurate T2 maps from highly undersampled FSE data. The proposed approach referred to as *DIrect REconstruction of Principal COmponent coefficient Maps* (DIREPCOM) employs sparsity constraints in both the spatial and temporal dimensions to produces accurate T2 maps.

Theory: Let **ρ** and $\mathbf{T_2}$ denote the proton-density and the T2 maps, respectively. Assuming a single exponential T2 decay, the signal model is given by $F_j(\boldsymbol{\rho}, \boldsymbol{T_2}, TE_j) = FT_j(\boldsymbol{\rho} \cdot e^{-TE_j/\boldsymbol{T_2}})$, where TE_j denotes the j[th] echo time, and FT_j is the forward Fourier Transform for given k-space locations at TE_j. In [3], an iterative algorithm for reconstructing T2 maps was formulated:

$$\arg\min_{\boldsymbol{\rho},\boldsymbol{T_2}} \sum_j \left\| F_j(\boldsymbol{\rho}, \boldsymbol{T_2}, TE_j) - \boldsymbol{K_j} \right\|^2 + \sum_i \lambda_i P_i(\boldsymbol{\rho}, \boldsymbol{T_2}), \qquad (Eq.1)$$

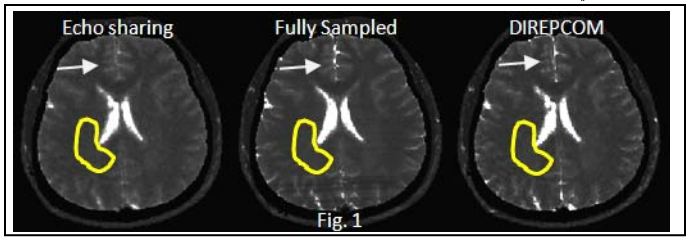

Fig. 1

where $\boldsymbol{K_j}$ denotes the measured k-space data at TE_j, P_i are penalty functions, and λ_i are regularization parameters. While the algorithm in [3] can obtain T2 maps from highly undersampled measurements, the non-linearity of the cost function introduced by the exponential term leads to sensitivity to noise and long reconstruction times. In addition, since the problem is non-convex, the algorithm can be trapped in local minima. In [4], Doneva et al illustrated that Principal Components (PCs) of a non-linear model can be calculated and used for sparsity within the CS framework. In this work we removed the non-linearity related to T2 estimation using Principal Component Analysis and combined it with CS to jointly exploit the spatial and temporal dependencies of the object.

The first step in the proposed technique is to calculate the PCs of exponential T2 decay curves for a given T2 range and echo time points by Singular Value Decomposition (SVD). An exponential T2 decay can be expressed as a weighted sum of these PCs. Let **B** denote the matrix whose columns are the PCs, the dimensions of **B** are N×N where N is the number of echoes (i.e. echo train length). Since the first few PCs are usually sufficient to accurately represent the signal, a truncated N×L matrix $\widehat{\boldsymbol{B}}$ can be formed by selecting the first L columns of **B**. Let **M** denote the matrix of PC coefficients. The dimensions of this matrix are the number of pixels in the image by the number of PCs after truncation (L). Let $\mathbf{M_i}$ denote the i[th] column of matrix **M** and $\widehat{\boldsymbol{B}}_j$ denote the j[th] row of matrix $\widehat{\boldsymbol{B}}$. The new optimization problem can now be stated as:

$$\arg\min_{\boldsymbol{M}} \sum_j \left\| FT_j(\boldsymbol{M}\widehat{\boldsymbol{B}}_j) - \boldsymbol{K_j} \right\|^2 + \sum_{i=1}^{L} \lambda_i TV(\boldsymbol{M_i}) + \sum_{i=1}^{L} \gamma_i |XFM(\boldsymbol{M_i})|. \qquad \text{(Eq.2)}$$

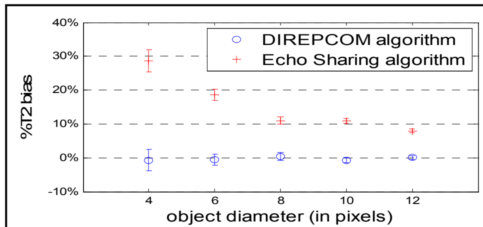

Fig. 2

In this equation, FT_j is the same as in *Eq.1*, *TV( )* denotes total variation, and *XFM( )* denotes a sparsifying transform (such as wavelets), and λ_i and γ_i are regularization parameters. Note that the non-linearity has been removed and the problem is now convex. In addition, by including sparsifying penalty functions on PC coefficient maps, both spatial and temporal dependencies can be exploited. T2 maps can then be derived from the matrix **M**.

Methods: Exponential decay curves at given TE's with T2 values from 50 ms to 500 ms equispaced by 1 ms were used as the training set. Eight TE values ranging from 8 ms to 64 ms equispaced by 8 ms were used to generate 8 PCs using SVD. Numerical simulations showed the 3 PCs with the largest singular values were sufficient for T2 estimation with absolute errors less than 0.1 ms. Hence only these 3 PCs are used in all reconstructions. *In vivo* data were acquired at 1.5T on a GE scanner using a radial FSE sequence with a single-channel brain coil (ETL=8, echo spacing=8.2 ms, TR=1s, 256 ×256, yielding a total of 32 k-space lines per TE). Gold standard data were acquired using the radial FSE method with 256 k-space lines per TE. For quantitative comparisons a numerical phantom consisting of circles of varying diameters with T2=100 ms was generated and used in simulations. K-space data for the phantom was generated analytically (parameters are the same as *in vivo* experiment) and i.i.d Gaussian noise was added in k-space to give an SNR comparable to *in vivo* data. The optimization problem was solved using a conjugate gradient algorithm and the regularization parameters were determined empirically. For comparison, T2 maps were also obtained using the echo sharing technique from [1].

Results: Figure 1 shows T2 maps reconstructed by the echo sharing and the DIREPCOM methods using the same undersampled radial FSE data. The figure also includes the gold standard T2 map. The mean T2 of the ROI highlighted in yellow from left to right are: 91.1ms, 91.8ms, 91.3ms indicating that for this larger ROI both algorithms yield T2's comparable to the gold standard. However, the DIREPCOM T2 map has better contrast in the small structures (indicated by the arrows) compared to the echo sharing T2 map, where similar structures are more blurred. Note that the amount of data used in the DIREPCOM T2 map here is only 1/8 of the amount used to generate the gold standard.

To further study the performance of DIREPCOM for T2 estimation of small structures we conducted simulations with objects of varying diameters. Figure 2 shows an error bar plot of the T2 estimates for the DIREPCOM and echo sharing algorithms from 20 noise realizations. The errors of the DIREPCOM estimates are significantly lower even for structures as small as 4 and 6 pixels in diameter. In these small structures the T2 bias obtained with echo sharing can reach 20-30% which is consistent with what was reported in [1].

Conclusions: In this work we proposed a novel algorithm to yield fast, robust and accurate T2 estimates from undersampled data. With this method T2 can be estimated from fast scans where only limited data can be acquired (such as abdominal breath-hold scans). While this proposed technique has been illustrated for T2 estimation, the methodology can be adapted to the estimation of other MR parameters.

Acknowledgement: NIH grants: CA099074 and HL085385 and the Alliance Beverage Award. **References: :** [1] Altbach et al, *MRM* 54:549, 2005, [2] Lustig M et al, MRM, 58: 1182, 2007. [3] Huang C, et al. Proc. ISMRM 17, 2740, 2009. [4] Doneva M, et al. ISMRM 17, 2812, 2009.