COMPRESSED SENSING FOR SPARSE MAGNETIC RESONANCE SPECTROSCOPY

X. Qu[1], X. Cao[2], D. Guo[1], and Z. Chen[3]

[1]Department of Communication Engineering, Xiamen University, Xiamen, Fujian, China, People's Republic of, [2]School Of Software, Shanghai Jiao Tong University, Shanghai, China, People's Republic of, [3]Department of Physics, Xiamen University, Xiamen, Fujian, China, People's Republic of

Introduction: Multidimensional magnetic resonance spectroscopy (MRS) can provide additional information at the expense of longer acquisition time than 1D MRS. Assuming 2D MRS is sparse in wavelet domain, Iddo[1] first introduced compressed sensing (CS) [2,3] to reconstruct multidimensional MRS from partial and random free induction decay (FID) data. However, the darkness in 1D MRS derives from the discrete nature of chemical groups [4]. Significant peaks in these MRS takes up partial location of the full MRS while the rest locations own very small or even no peaks. This type of MRS can be considered to be sparse itself, named sparse MRS. In the concept of sparsity and coherence for CS[5], we will demonstrate that wavelet is unnecessary to sparsify sparse MRS, and it makes the reconstructed MRS even worse than using identity matrix. Furthermore, a l_p quasi-norm compressed sensing reconstruction is employed to improve the quality of reconstruction.

Methods: For a signal $\mathbf{x}$ that can be represented by S-term non-zero entries with vector $\boldsymbol{\alpha}$ in basis $\boldsymbol{\Psi}$, $\mathbf{x}$ can be recovered by solving l_1 norm optimization when number of measurements M satisfies $M \geq C \cdot \mu^2(\boldsymbol{\Phi}, \boldsymbol{\Psi}) \cdot S \cdot \log N$ where μ stands for the coherence between sensing matrix $\boldsymbol{\Phi}$ and basis matrix $\boldsymbol{\Psi}$ [4]. It implies that the number of required measurements is proportional to the number of nonzero entries in $\boldsymbol{\alpha}$ and the square of coherence μ. Although wavelet is a general transform to sparsify MRS, those MRS from chemical groups carry large peaks in only small portion of locations. These MRS can be viewed as sparse MRS which means spectroscopy is sparse in identity matrix **I**. In fact, Stern *et al* proposed to directly do iterative thresholding on the spectroscopy to recover the truncated 1D NMR spectroscopy [6] which implicitly supports this idea. In addition, pioneer works on CS theory pointed out that the coherence between Fourier and wavelet is larger than that of Fourier and time because time-frequency is with the minimal coherence [5]. At this point there is no need to do the wavelet transform on the spectroscopy which is sparse in identity matrix **I**. In addition, Rick Chartrand [7] proposed to replace l_1 norm with l_p quasi-norm to reconstruct the signal with fewer measurements than l_1 requires. Thus, we propose to employ compressed sensing to reconstruct multidimensional sparse MRS from partial FID data as follows:

$$\hat{\mathbf{x}} = \min_{\mathbf{x}} \|\mathbf{x}\|_p \quad s.t. \left\|\mathbf{y} - \Theta \mathbf{F}^T \mathbf{I} \mathbf{x}\right\|_2 \leq C_0$$

where **y** is the partial and random FID data, Θ is a random sampling operator that determines the phase lines to be randomly and partially acquired, $\mathbf{F}^T$ denotes the inverse 2D Fourier transform and C_0 controls the data consistency.

Results: Measuring the decay of coefficients' magnitude in a sparsifying transform domain is a simple way to test the sparsity. Fig.1(b) shows that decay of identity matrix decreases faster than that of wavelet. It means this type of spectroscopy is sparser in identity matrix domain than in wavelet domain. For the random sampling mask shown in Fig.1(c), the mutual coherence between wavelet and $\Theta\mathbf{F}^\mathbf{T}$ is 0.99 and mutual coherence between identity matrix and $\Theta\mathbf{F}^\mathbf{T}$ is 0.25. Fig.1 (e) and (f) show that identity matrix can recover much more peaks than wavelet does. One can also observe pseudo peaks in the spectra of indirection dimension using wavelet-based CS while identity matrix-based CS does not. Compared with Fig.1 (e), Fig.1(g) shows replacing l_1 norm with l_p norm can further improve the reconstruction quality. We introduce normalized mean square error (NMSE) defined as $NMSE = \|\tilde{\mathbf{x}} - \hat{\mathbf{x}}\|_2 / \|\tilde{\mathbf{x}}\|_2$ to evaluate the difference between fully sampled spectra $\tilde{\mathbf{x}}$ and reconstructed spectra $\hat{\mathbf{x}}$ with partial FID data.

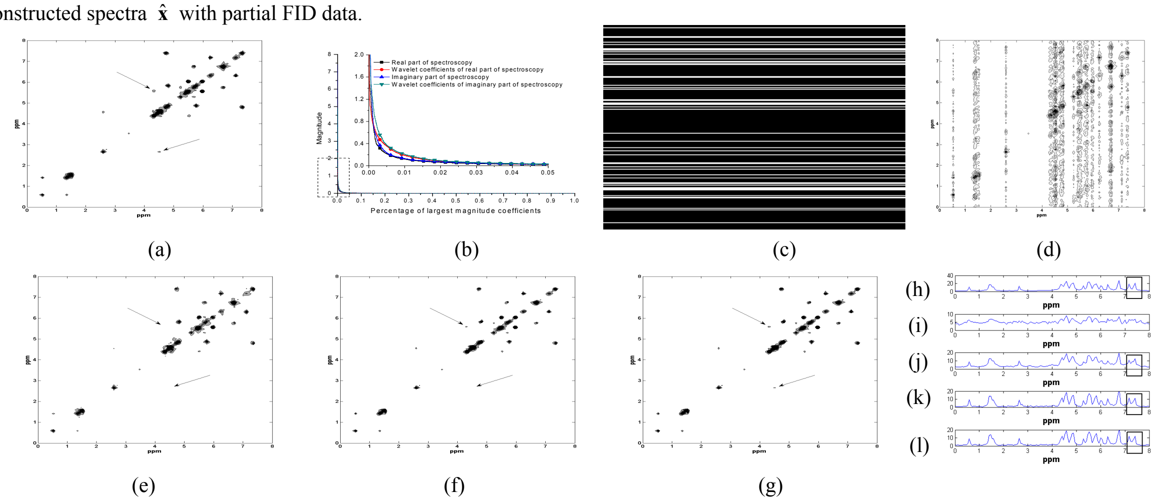

Fig.1 Reconstruction of ^{1}H-^{1}H spectroscopy.(a) fully-sampled MRS (b) decay of coefficients' magnitude , (c) 20% phase lines are randomly sampled, (d)-(g) are reconstructed MRS with zero-filling(NMSE=0.83) ,wavelet-based CS with l_1 norm(NMSE=0.31), identity matrix-based CS with l_1 norm(NMSE=0.25), identity matrix-based CS with l_p (p=0.5) norm (NMSE=0.21), (h)-(l) are the spectra of indirect dimension corresponding to (a),(d),(e),(f),(g), respectively .

Conclusions and Discussion: For sparse MRS, wavelet is not necessary and even worsen the reconstructed spectra. With the l_p quasi-norm, quality of reconstructed spectra can be further improved. However, how to define the meaningless peaks depends on applications. A qualitative analysis of sparse MRS is needed in order to satisfy the requirement of CS. Method proposed in this paper can extend to higher dimension MRS than 2D.

Acknowledgement: This work was partially supported by NNSF of China under Grants (10774125 and 10605019)

References:

[1] Iddo D., Eurasip J Adv Sig Pr 2007, Article ID 20248

[2] Candès E.J. et al.. IEEE T Information Theory 2006;52:489-509.

[3] Donoho D. L. IEEE T Information Theory 2006; 52:1289-1306.

[4] Matsuki Y. et al.. JACS 2009; 131:4648-4656

[5] Candès E.J. et al.. Inverse Problem 2006; 23:969-985.

[6] Stern A.S, et al.. JMR 2007;188: 295-300

[7] Chartrand R., IEEE Signal Proc Let 2007;14:707-710