A stochastic framework for improving the accuracy of PIESNO

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INTRODUCTION Probabilistic Identification and Estimation of Noise (PIESNO) is a recent computational framework proposed in [1] to simultaneously identify noise-only pixels in magnitude-reconstructed MR images and estimate the noise level (commonly known as the Gaussian noise standard deviation or variance) in these images. PIESNO works on a typical data structure as depicted in Fig. 1. Interests in the estimation of the noise level in magnitude-reconstructed MR images, e.g.,[1-7], have been increasing since the works of Edelstein et al. [2] and Henkelman [3] because many MR applications, e.g., [5-6, 10-12], are critically dependent the accurate estimate of the noise level. Although PIESNO is a highly efficient technique, its efficiency comes at a slight cost to the accuracy in estimating of the noise level. Here, we outline a simple stochastic framework to improve the accuracy of PIESNO.

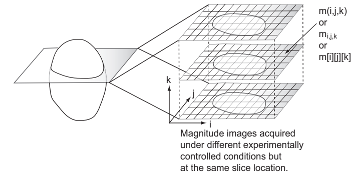

Fig. 1. PIESNO is specifically designed to take advantage of the data structure shown above. Many MRI protocols produce this type of data structure; notably, fMRI and diffusion MRI.

METHODS AND RESULTS Magnitude-reconstructed MR signals, *m*, obtained from an *N*-receiver-coil MRI system [13] follow a nonCentral Chi distribution of *2N* degrees of freedom [14,15] and the PDF of noise in magnitude MR images is the Central Chi distribution of the following form:

$$(m^{2N-1} / 2^{N-1} \sigma_g^{2N} (N-1)!) \exp(-m^2 / 2\sigma_g^2) \quad . \qquad [1]$$

By making a change of variables in the PDF of noise, it can be shown that the new variable, which is the ratio of the square of the magnitude signal to $2\sigma_g^2$, follows a particular type of the Gamma PDF, i.e., Gamma(*N*,*1*) [16]. Due to the reproductive property of the Gamma distribution, the arithmetic mean, denoted by *s*, of *K* independent measurements of the new variable is again a Gamma random variable of a different type, i.e., Gamma(*NK*,*1/K*). In brief, the arithmetic mean, *s*, is expressed as

$$s_{i,j,k} = (1/2K\sigma_g^2)\sum_{k}^{K} m_{i,j,k}^2 \quad . \qquad [2]$$

The identification of noise is carried out probabilistically by specifying the lower and upper threshold values (respectively, $\lambda_-(\alpha, N, K)$ and $\lambda_+(\alpha, N, K)$) of *s* for a given probability level α, which can be computed readily from the cumulative distribution function (CDF) of *s*. The estimation of the standard deviation of Gaussian noise is based the optimal method, which is given by:

$$\tilde{\sigma}_g = q_\alpha / \sqrt{2P_s^{-1}(\alpha \mid N,1)} \quad , \qquad [3]$$

where q_α is the optimal quantile of the selected magnitude signals whose arithmetic mean falls within the lower and upper threshold values and P_s^{-1} is the inverse CDF of s. The order of the quantile can be obtained through numerical optimization and is a function of *N*, see [1].

The criterion in which the magnitude signals are identified as noise-only measurements is carried out through the selection of appropriate s as determined by the following inequalities: $\lambda_-(\alpha, N, K) \le s \le \lambda_+(\alpha, N, K)$. This criterion was chosen for expediency in terms of computational efficiency and theoretical simplicity rather than for accuracy. Although a strictly theoretical approach to determine the exact level of bias in the estimate of noise level through the use of this criterion and Eq.[3] seems to be intractable, we believe it is still worthwhile to share the present stochastic framework for determining the level of bias. The goal of this framework is therefore to determine the level bias in [3] so that the more accurate estimate, σ, can be obtained by incorporating an additive term that is a function of α, *N*, *K* , i.e.,

$$\sigma = \tilde{\sigma}_g + \varepsilon(\alpha, N, K) \quad . \qquad [4]$$

Based on our numerical experience, the additive term turns out to be independent of the noise level used to generate the random signals.

Here, we outline this stochastic framework in a step-by-step procedure:

1. Choose a desired set of parameters, α, *N*, *K*, the matrix dimension of the data structure, and the number of Monte Carlo simulation runs, J.
2. For each run, create the random Central Chi-distributed signals, which depend on *N* and the noise level, and populate the data structure with these random signals.
3. Perform the optimal PIESNO to obtain $\tilde{\sigma}_g$.
4. Repeat J number of runs and collect J number of these estimates, $\tilde{\sigma}_g$, and determine their mean value. The difference is then used as the additive term.

It is clear then that the precision of the additive term will depend on the number of simulation runs.

DISCUSSION & CONCLUSION The goal of the proposed stochastic framework is improve the accuracy of PIESNO in estimating the noise level in magnitude-reconstructed MR images. We found that the additive term is independent of the noise level used to generate the random MR magnitude noise. The level of bias is relatively small. In terms of the percent relative error for the case of *N*=1, it is about 1.1% for α=0.1 and K=6, 0.4% for α=0.1 and K=12, 0.2% for α=0.1 and K=30. The relative error decreases as the numerical value of α decreases. The present stochastic framework is, to the best of our knowledge, the first computational framework in addressing the issue of determining the bias level in the estimate of noise level computed from PIESNO. Since PIESNO is a general framework with diverse applications in imaging science, it is our hope that the present stochastic framework may be of help to users of PIESNO. PIESNO software can be found in [17].

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