Simplified Signal Equations for Spoiled Gradient Echo MRI

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INTRODUCTION: This work presents simplified signal equations for spoiled gradient echo (SPGR) imaging that, to the authors' knowledge, has not been published. The framework introduces an exponential time constant TA, which reflects magnetization loss from the rf pulse. This framework is then used for to consider image SNR and T1 contrast.

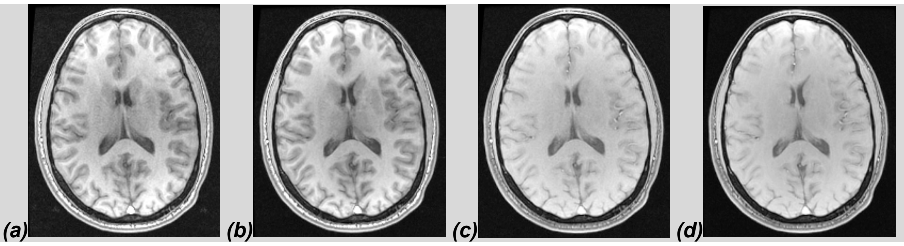
THEORY: Image SNR can be approximated as the product of (SNR/spin)*(spins/voxel), the first term derived from spin physics, and the second representing voxel volume (resolution) and proton density. This abstract focuses on the first term, which will be called SNR_{spin}. This work also sets many constant terms to one without loss of generality. The longitudinal magnetization loss each TR from a flip angle α is $\cos(\alpha)$; this magnetization loss, along with T1 recovery, affects the signal and determines T1 contrast in an image, and can be equated to an exponential signal loss using a time constant TA, i.e.

$$\cos(\alpha) = e^{-TR/TA} \qquad [1]$$

If one varies α and TR according to Eq. [1] in order to keep a constant TA, T1 contrast remains equivalent; this is illustrated in Fig. 1.

Fig. 1. Collected 3D SPGR images (GE 3T Signa) illustrating that T1 contrast is the same across TR's using Eq. [1] for fixed TA.

	(a)	(b)	(c)	(d)
TA	160ms	160ms	1000ms	1000ms
TR	12ms	28ms	12ms	28ms
α	22˚	33˚	9˚	13˚



Combining Eq. [1] with the signal equation for SPGR, assuming TR << T1 and TR<<TA, one obtains a spin signal s of

$$s = \sin(\alpha)\frac{1-e^{-TR/T1}}{1-\cos(\alpha)e^{-TR/T1}}e^{-t/T2*} = \sqrt{1-e^{-2TR/TA}}\,\frac{1-e^{-TR/T1}}{1-e^{-TR/TA}e^{-TR/T1}}e^{-t/T2*} \approx \frac{\sqrt{2\ TR\ TA}}{T1+TA}e^{-t/T2*}. \qquad [2]$$

The SNR_{spin} of an image is obtained by multiplying Eq. [2] by the sampling efficiency $(ADC/TR)^{-1/2}$, the square root of scan time τ, and replacing $e^{-t/T2*}$ with S_{ave}, which accounts for SNR loss both from T2* decay during the sampling duration ADC as well from the sampling density correction weights applied for image reconstruction, i.e.

$$SNR_{spin} \approx \frac{\sqrt{2\ TR\ TA}}{T1+TA}\sqrt{\frac{ADC}{TR}}\sqrt{\tau}\,S_{ave} = \frac{\sqrt{2\ TA}}{T1+TA}\sqrt{ADC}\sqrt{\tau}\,S_{ave}, \qquad [3]$$

where

$$S_{ave} = \frac{\sum\left(w_n\ \mathrm{e}^{-tn/T2*}\right)}{\sqrt{N}\sqrt{\sum w_n^2}}, \qquad [4]$$

w_n is the sampling-density-correction weight of the n^th data point (N total) in k-space, and tn is the time after excitation that it was sampled. S_{ave} is bounded by 0 and 1, and maximized for $w_n = e^{-tn/T2*}$ (matched filtering); note that w_n is primarily chosen to weight k-space in order to create uniform data representation across measured k-space (e.g., ref (4)). An interesting consequence of Eq. [3] is that for fixed resolution and contrast (fixed T1, TA), image SNR (normalized by $\tau^{-1/2}$) depends only on ADC and S_{ave}, i.e.

$$\left.\frac{\mathrm{SNR}_{\mathrm{spin}}}{\sqrt{\tau}}\right|_{fixedT1,TA} \propto \sqrt{ADC}\ S_{ave}. \qquad [6]$$

Methods such a signal averaging, partial Fourier imaging, and parallel imaging can be used to change τ, but do not significantly change Eq. [6] (other than SNR_{spin} reduction from the G factor in parallel imaging) if ADC << T2*. For longer ADC's, S_{ave} may change, and must be analyzed for the given sequence and choice of w_n, taking into account the effect of T2* decay on voxel shape and volume.

The (differential) T1 contrast to noise ratio CNR at a given T1 is derived from Eq. [3], i.e.

$$CNR = \frac{dSNR_{spin}}{dT1} \approx -\frac{\sqrt{2\ TA}}{(T1+TA)^2}\sqrt{ADC}\sqrt{\tau}\ S_{ave}. \qquad [4]$$

The Ernst angle is achieved for TA=T1, maximizing SNR_{spin}, while CNR is maximized by TA = T1 / 3.

CONCLUSION: This work illustrates a convenient form of writing and approximating the signal equations for SPGR, using the variable TA to replace TR and flip angle, and gives consistent results with other work, e.g. refs (1-3). In this form, other imaging parameters are easily analyzed for fixed contrast (i.e. fixed TA). Equation [6] illustrates that lengthening ADC is an important tool in enhancing image SNR, when suitable methods for the effects of off-resonance phase, eddy currents, and concomitant fields are available.

REFERENCES: [1] Pelc, Mag Res Med 1993, 29(5); 695-9. [2] Busse, Mag Res Med 2005, 53(4); 977-80. [3] Neelavalli, Haacke, Mag Res Imaging 2007, 25; 1397-1401. [4] Johnson, Pipe, Mag Res Med 2009, 61(2); 439-47.