MR Gradient Estimation Using a Linear Time Invariant Model

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Introduction: MRI applications such as functional MRI and dynamic imaging benefit from fast imaging trajectories such as spirals and 3D cones [1]. One complication of fast imaging with time-varying gradients is that the actual *k*-space trajectories are likely to deviate from the theoretical case due to gradient system imperfections. Reconstructing images with gradients measured on the scanner provides an accurate but time-consuming solution. Because the main effects of these imperfections can be modeled as a delay, methods exist to calculate a single system delay for each axis and use this information to approximate the actual trajectories [2,3]. However, delay values depend on the gradient waveform, as shown for a 3D cones trajectory [4]. A single delay value may not be able to accurately approximate a range of gradient waveforms. In this work, we present a method to correct for *k*-space trajectory deviations of multiple trajectories with a single characterization by describing the gradient system with a linear time invariant (LTI) model. The results are compared to current methods.

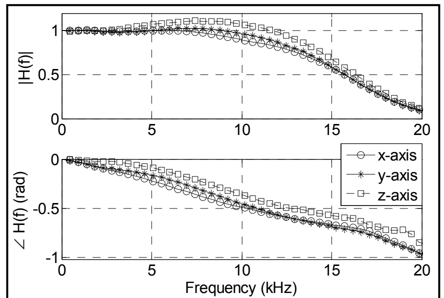

Figure 1: System Response

Theory: Current methods seek to find a system delay for each axis assuming that all frequencies in the MR gradient system experience the same delay. This is unlikely, due to system imperfections. Assuming linearity holds [5], an LTI model would allow the nonlinear phase of the gradient system - consequently, a range of delay values - to be incorporated into gradient waveform approximations. In previous work [6], a set of sinusoids were measured on an MR scanner and the response of each was used to generate an LTI model. In this work, a simpler approach was used to calculate the system response by dividing the Fourier transform of one high bandwidth gradient waveform by its theoretical transform. Imaging gradients were then approximated by convolution with the impulse response.

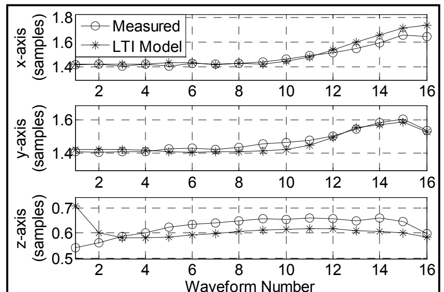

Figure 2: Cones Trajectory Gradient Delays

Method: Experiments were run on a 1.5 T GE Signa Excite Scanner with cardiac resonator module (CRM) gradients with maximum amplitude 40 mT/m and slew rate 150 mT/m/ms. All sequences used a ±125 kHz receiver bandwidth. Gradients were measured using a spin echo sequence with slice selection and readout in the same direction [4]. Sequence parameters were: TR = 200 ms, four 2 mm slices at ±3, ±1 cm from isocenter, 8 averages/slice, and. Depending on the measured gradient waveform duration, TE values ranged from 19 to 30 ms. A 5 mT/m, 72 µs triangle gradient waveform was used as an approximation of an impulse to calculate the system response. This was a ±16.8 kHz bandwidth signal, which was helpful for tracking the gradient while operating near the slew rate limit. Spiral and 3D cones sequences were tested. The design parameters for the spiral trajectory were: 16 interleaves, 20x20 cm^2 field of view (FOV), 1x1 mm^2 resolution. The maximum amplitude and slew rate were 29 mT/m and 140 mT/m/ms. An axial slice was scanned using: TE/TR = 10/500 ms, slice thickness 5 mm, and a 8.7 ms readout duration. The 3D cones trajectory design parameters were: 16 waveforms, 24x24x24 cm^3 FOV, 1x1x1 mm^3 resolution, and 6 ms readouts. The maximum amplitude and slew rate were 40 mT/m and 150 mT/m/ms. Imaging was done using a spoiled gradient echo sequence with: TE/TR = 1/10.6 ms, 30° flip angle, and obtained with 10 dummy cycles. Images were reconstructed with 1) theoretical trajectories, 2) trajectories adjusted by the average measured delay for a given axis, 3) trajectories convolved with the LTI model impulse response, and 4) measured trajectories.

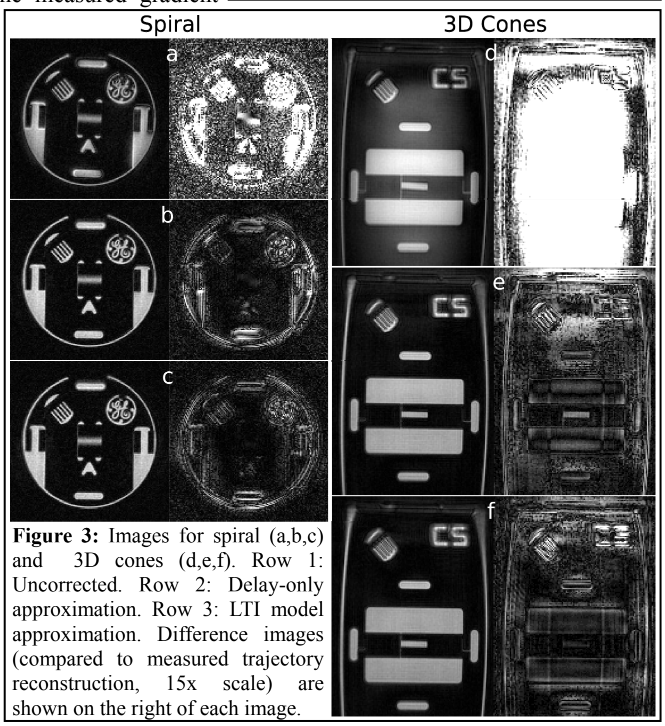

Figure 3: Images for spiral (a,b,c) and 3D cones (d,e,f). Row 1: Uncorrected. Row 2: Delay-only approximation. Row 3: LTI model approximation. Difference images (compared to measured trajectory reconstruction, 15x scale) are shown on the right of each image.

Results/Discussion: Figure 1 shows the calculated system response for the gradient system based on the triangle waveform input. As expected, the phase responses for each axis were not exactly linear. This shows that the proposed model can capture a range of delay values for each gradient axis. Fig. 2 gives a plot of the measured delays of the gradient waveforms for each waveform set of the cones sequence. The measured delays for the spiral sequence (not shown) remained essentially constant at 1.4 samples for the x and y-axes. However, for the cones, the delays were 1.48±0.13, 1.47±0.1, 0.63±0.06 samples for the x, y, and z-axes. Fig. 2 also shows the ability of the LTI model to generate waveforms with similar delays to those of the measured waveforms, although the match is not as accurate for the z-axis. Fig. 3 shows reconstructed axial spiral images and reformatted coronal 3D cones images. Both methods achieve good correction of rotation and shading artifacts. Using images reconstructed with measured trajectories as the standard (not shown), difference images were plotted. For the spiral sequence, the normalized root mean square error (NRMSE) was 9.92% and 8.16% for the delay-only and LTI estimated trajectories. For the cones sequence, the NRMSE was 11.2% and 8.6% respectively. Compared to using the delay-only method, an LTI model provides an 18% and 23% improvement in image error reduction for the spiral and 3D cones sequences, respectively. This shows the potential of a single LTI model to accurately approximate *k*-space trajectories achieved on the scanner for a range of gradient waveforms.

References: [1] Gurney et al., MRM, 55:575-582, 2006. [2] Tan et al., MRM, 61:1396-1404, 2009. [3] Atkinson et al., MRM, 62:532-537, 2009. [4] Gurney, PhD Thesis, Stanford University, 2007. [5] Brodsky et al., ISMRM, 2799, 2009. [6] Kerr, PhD Thesis, Stanford University, 1998.