## Theoretical Sensitivities of SWIFT and the Ideal Sequence (delta pulse-acquire) for Ultra-short T2

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**Introduction:** The ideal sequence for measuring ultra-short  $T_2$  spins comprises a series of infinitely short excitation pulses ( $\delta$ -pulses) followed immediately by FID acquisition, with no delay in between. However, this sequence cannot be practically realized since RF pulses have finite duration  $(T_p)$  and time is required for scanner electronics to switch between transmit and receive modes. Furthermore, off-resonance spins are affected differently when using conventional hard pulses having finite duration. A relatively new method known as SWIFT, sweep imaging with Fourier transformation [1], comes close to offering many of the desirable features of the  $\delta$ -pulse sequence. The purpose of this work was to provide a theoretical framework and computer simulations to compare SWIFT with the  $\delta$ - and hard-pulse sequences in steady state.

**Theory:** The signal energy for a spoiled RF pulse sequence is given by  $S^2(\theta) = \frac{N_A}{N_{\text{max}}} \frac{M_0^2 T_2 \sin^2(\theta)(1-E_1)^2}{2T_A(1-\cos(\theta)E_1)^2} (1-E_2^2)$ , where  $\theta$  is the flip angle,  $N_A$  is the

number of acquired points,  $N_{\text{max}}$  is the maximum number of acquired points possible,  $E_1 = \exp(-T_R/T_1)$ ,  $E_2 = \exp(-T_A/T_2)$ ,  $T_R$  is the repetition time, and  $T_A$  is the acquisition time [2]. For a given  $T_1$  and  $T_2$  there is a value of  $\theta$  producing maximum  $S^2$  which is predicted by the Ernst equation,  $\cos(\theta_{\text{max}}) = E_1$ , where  $\theta_{\text{max}}$  is the Ernst angle.

**Method**: The NMR system was modeled by using a Bloch simulator. The Bloch equations were numerically solved using the Runge-Kutta method for three different types of sequences: 1) a  $\delta$ -pulse, 2) a hard pulse, and 3) SWIFT. The  $\delta$ -pulse sequence used  $T_{\rm R}$  = 4.045 ms and  $T_{\rm A}$  = 4 ms. The SWIFT sequence used  $T_{\rm p}$  = 4 ms with 45 µs post-acquisition delay (giving  $T_{\rm R}$  = 4.045 ms). For all simulations the acquisition bandwidth was 127 kHz. An ideal SWIFT sequence has simultaneous excitation and acquisition. However due to hardware limitations, the signal must be acquired in gaps within the pulse sequence. Both gapped and un-gapped simulations of SWIFT were done.  $T_{\rm p}$  of the hard pulse was calibrated so that the pulse amplitude corresponded with the maximum pulse amplitude of the SWIFT pulse for a given  $\theta$ , resulting in  $T_{\rm p} \le 45$  µs. They also had  $T_{\rm A}$  = 4 ms. For all simulations,  $T_1 = T_2$ , which is indicated by  $T_{1,2}$ . The effect of having a resonance offset was also investigated.

**Results and Discussion:** Figure 1 shows  $\theta_{\text{max}}$  plotted as a function of  $1/T_{1,2}$  for each pulse sequence. It can be seen that the  $\delta$ -pulse sequence, hard pulse sequence on-resonance, and SWIFT sequence on- and off-resonance (10 kHz offset) exhibit the predicted  $\theta_{\text{max}}$ . However, the hard pulse off-resonance (5 and 10 kHz offset) show a higher  $\theta_{\text{max}}$  than the equation predicts. Figure 2 shows a plot of the  $1/T_{1,2}$  values at which maximum signal energy ( $S_{\text{max}}^2$ ) occurs as a function of  $\theta$ . For a given  $\theta$ , SWIFT achieves  $S_{\text{max}}^2$  at a shorter  $T_{1,2}$  value than the  $\delta$ -pulse sequence for both resonance frequencies. It is notable that  $S_{\text{max}}^2$  occurs at longer  $T_{1,2}$  values as a function of resonance offset when using hard pulses. The graph is useful since it allows determination of the value of  $\theta$  yielding highest  $S^2$  for given spins of interest with a certain  $T_{1,2}$ . Also, the value of  $\theta$  at which  $S_{\text{max}}^2$  occurs is smallest with SWIFT, which is advantageous when RF power is limiting. Figure 3 shows a plot of  $S^2$  normalized by the  $\delta$ -pulse signal energy ( $S_{\delta}^2$ ) produced with  $\theta = 20^{\circ}$ . It shows that with on-resonance hard pulses there is some signal loss for short  $T_{1,2}$ . In comparison, off-resonance hard pulses produce greater  $S^2$  from long  $T_{1,2}$  spins, but reduced  $S^2$  for shorter  $T_{1,2}$  spins. For long  $T_{1,2}$ , SWIFT produces reduced  $S^2/S_{\delta}^2$ , but increases toward unity as  $T_{1,2}$  gets shorter. Two different phenomena account for this behavior: 1) For SWIFT the excitation occurs approximately in the middle of the pulse so  $T_A$  is effectively halved. 2) The gapped SWIFT has  $N_A/N_{\text{max}} = 50\%$ , so  $S^2$  is also halved. However, making these two corrections in the Ernst energy equation yields good agreement. The off-resonance hard pulses also suffer from two different phenomena: 1) For short  $T_{1,2}$ , there is signal loss during the pulse. 2) The frequency offset causes the effective  $\theta$  to be reduced.

**Conclusions:** SWIFT appears to be a powerful new tool for MRI. The sensitivity of SWIFT is described by the Ernst equations, which can be used to determine sequence parameters to maximize signal energy for specific relaxation times. Furthermore, a flip angle can be determined so that an object with given  $T_{1,2}$  will have the greatest relative energy in the system. This flip angle will be smaller for SWIFT than for  $\delta$ - or hard-pulse sequences for a given  $T_{1,2}$ , which helps to reduce RF power requirements. This work demonstrates that, unlike the hard-pulse sequence, SWIFT is unaffected by resonance offsets (eg, B<sub>0</sub> inhomogeneity) and thus may be well suited for imaging superparamagnetic iron oxides and near metallic implants.



References:[1] Idiyatullin et al., J. Magn. Reson., 181 (2006), p. 342-349 [2] Ernst et al. Oxford Sci. Pub., 1987, p. 153 Grants: This research was supported by NIH BTRC grant #P41-RR008079