Reconstruction method for non-homogeneous magnetic fields using the Fractional Fourier Transform

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INTRODUCTION: In Magnetic Resonance Imaging (MRI), the distortions produced by field inhomogeneities increase with high B_0 fields and with long acquisition sequences such as echo–planar (EPI) and spiral imaging. Both passive and active shimming can be applied to reduce inhomogeneities, and some further corrections can be achieved with post processing techniques, e.g. linear correction and conjugate phase reconstruction methods [1][2]. However, these techniques lack of higher order corrections, require long processing times, or are heuristic in nature, so that their applicability is limited. In MRI, the signal generated by an object under a homogeneous field is related to its magnetization by the Fourier Transform (FT). We will here use the Fractional Fourier Transform (FrFT) [3] to relate the MR signal generated by an object under spatially–varying quadratic field with its magnetization. Using this relationship we will propose a reconstruction scheme to correct the distortions caused by quadratic field inhomogeneities. We present experiments with numerical and physical phantoms and with *in vivo* images to validate our claim.

THEORETICAL BACKGROUND: The FrFT is a generalization of the FT that can be thought as a one–parameter interpolation between the object and its Fourier transform. The parameter is commonly referred as the fractional order of the transform and it is denoted by α, where $\alpha = 0$ represents the image domain and $\alpha = \pi/2$ represents the Fourier domain. For simplicity, we will consider 1D MR signals and normalized gradients. In the case of quadratic inhomogeneities, the field deviation can be represented as $G(x,t) = G_0 x + p(x)$ with $p(x) = p_0 + p_1 x + p_2 x^2$. In this case the signal equation becomes:

$$s(t) = \int m(x) e^{-2\pi i (k(t)x + p(x)t)} dx = e^{-2\pi i p_0 t} \int m(x) e^{i\pi(-2p_2 t x^2 - 2(k(t)+p_1 t)x)} dx \qquad (1)$$

where $k(t) = G_0 t$ represents the measured Fourier coefficient in the homogeneous case. Using the change of variables $\cot\alpha(t) = -2p_2 t$ and $2\rho(t)\csc\alpha(t) = -2(k(t) + p_1 t)$, we can relate $s(t)$ to the FrFT of $m(x)$ by:

$$s(t) = e^{-2\pi i p_0 t} \int m(x) e^{i\pi(-2p_2 t x^2 - 2(k(t)+p_1 x)t)} dx = e^{-2p_0 t} \int m(x) e^{i\pi(\cot\alpha(t) x^2 + 2\csc\alpha(t)\rho(t)x)} dx \qquad (2)$$

$$s(t) = e^{-2\pi i p_0 t} e^{-i\pi \cot\alpha(t)\rho^2(t)} \sqrt{1 - i\cot\alpha(t)}\; F^{\alpha(t)}[m](\rho(t)) \qquad (3)$$

where $F^{\alpha}[m](\rho)$ represents the FrFT of fractional order α of the magnetization m measured at the fractional frequency ρ. If the parameters (p_1, p_2, p_3) describing the quadratic distortion are known, one can precisely locate the fractional order and frequency corresponding to the measured data at the time instant *t*. Thus, correcting the measured data by the appropriate scaling factors of Eq.3, the object $m(x)$ can be reconstructed from $s(t)$ by means of the discretization of the inverse of the FrFT.

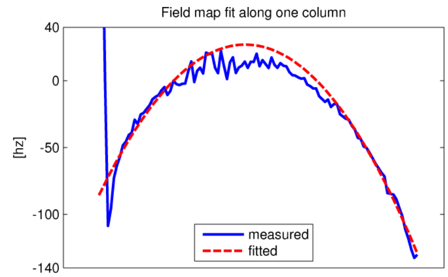

Figure 1

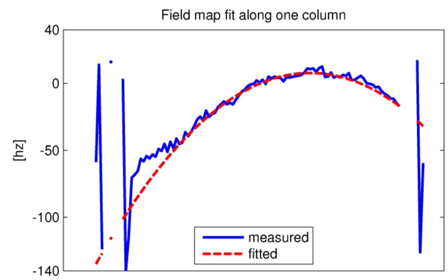

Figure 2

EXPERIMENTS: For the phantom studies, two data sets were used: (1) a numerical phantom consisting of 256×256 samples with Cartesian sampling; (2) data acquired on a cylindrical phantom consisting of 128×128 samples using a 2D FFE multi shot EPI sequence (FOV: 24×24cm, slice thickness 5mm, $\alpha = 23°$, $T_R = 650$ms, $T_E = 41$ms, NSA = 16, EPI factor 63, read–out time: 76ms) with a Q–body coil. For the *in vivo* experiment, a brain image was acquired using the same sequence used for the physical phantom, except from NSA = 8 and a quadrature head coil. To observe distorted images produced by field inhomogeneities, long read–out times were used. Also in each case, a fully sampled image was acquired as reference (FFE, $T_R = 14$ms and $T_E = 6.1$ms for phantoms and $T_R = 17$ms and $T_E = 6.2$ms for *in vivo* images). Moreover, on each experiment we measured the field map via phase differences between two acquired images [4]. A quadratic function was fitted to each field map within the boundaries of the object using a maximum likelihood method. The fitted function obtained on the cylindrical phantom was use as a perfect quadratic field to distort the numerical phantom (read–out time: 28ms). Phantoms and in-vivo data were reconstructed using the standard FT and our proposed method. For the FrFT reconstruction we used the fitted parameters to deduce the fractional order α and frequencies ρ.

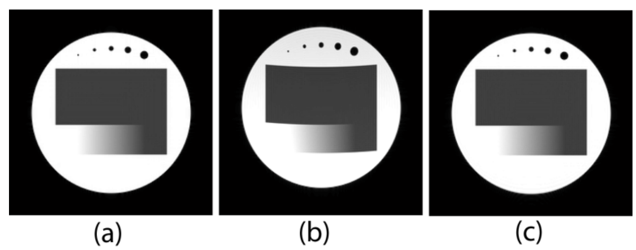

Figure 3

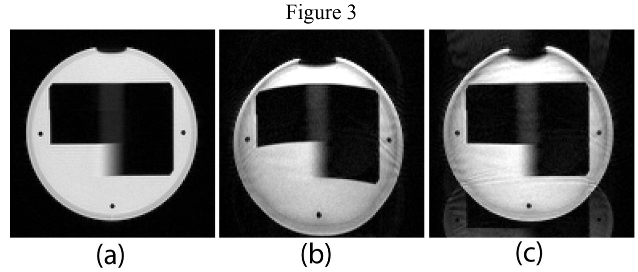

Figure 4

RESULTS: The quadratic distortions present in the field map can be well approximated by a quadratic function for both the physical phantom (Fig. 1) and the brain experiment (Fig. 2) (only one column is plotted). For the numerical phantom (Fig. 3), the proposed FrFT–based reconstruction method is able to correct (Fig. 3c) the geometric distortions present when a standard FT reconstruction is applied (Fig. 3b). This behavior is consistent to the physical phantom (Fig. 4) and the experiment with brain images (Fig. 5), even though artifacts are present on the FrFT reconstruction (Fig 4c and 5c). We attribute these artifacts to the fact that the distortion is not exactly quadratic as in the numerical phantom case. The bending and vertical displacement is noticeably seen in standard FT–reconstructed phantom images (Fig 3b and 4b) because of the straight line patterns in the phantom. To facilitate the observation of the same distortions in the brain images, contour lines computed at the boundaries of the reference image (Fig 5a) have been superimposed on the standard FT (Fig 5b) and the FrFT (Fig 5c) reconstructed images, demonstrating the quality of the corrections of our proposed method.

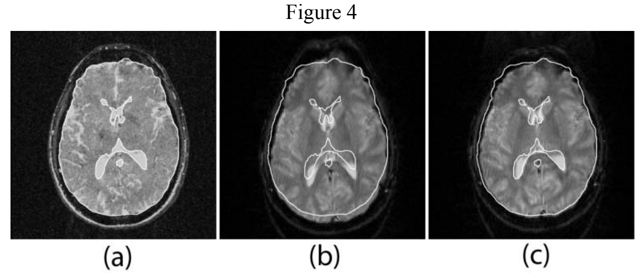

Figure 5

CONCLUSION: The FrFT offers an effective method to reconstruct MR images acquired with field inhomogeneities that are well approximated by a quadratic function. The method is able to correct the geometric distortions present on the reconstructions performed with a standard FT. This fact suggests that there could be a framework which would allow native MR reconstruction using quadratic fields.

REFERENCES: [1] Irarrazaval *et al.,* MRM, 35(2), 1996, [2] Noll *et al*., TMI, 24(3), 2005, [3] Ozaktas *et al*., Wiley, 2001, [4] Schneider et al., MRM, 18(2), 1991.