FREBAS Domain Super-Resolution Reconstruction of MR Images

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Introduction: Super-resolution is a method of generating images beyond the limit of the resolution. Recently, a method by which to realize super-resolution by a technique that performs registration by a sub-pixel unit from several pieces of an image has been reported[1]. Gerchberg-Papoulis (GP) method[2] is known to realize super-resolution from a single image and signal, however, spatial resolution will not be improved well when it is based on the Fourier transform. On the other hand, GP method involving convolution integral can expand the signal band easily and the resultant image has higher resolution. In this study, we investigated the super-resolution of images using FREBAS transform that can be considered as a kind of multi-resolution image analysis based on convolution integral. Improvement of resolution on the image space with reference to the scaling parameter of FREBAS transform is examined.

Theory: When image down-scaling is performed by using the two algorithms of discrete Fresnel transform formula, the alias signal that is produced when computing the Fresnel transform signal is separated in the reconstructed image domain, and the equivalent band splitting of the Fresnel transformed signal is performed in the image domain (FREBAS)[3]. This band splitting of the Fresnel transformed signal domain can be interpreted as an image data being analyzed by convoluting imaged data with sinc function having different modulation indices. Each expanded image in the FREBSAS domain can be described equivalently as a convolution integral written as:

$$i_m = I(x)e^{-jcx^2} * g(x), \qquad g(x) = \mathrm{sinc}\,(2cXx)e^{j2cmXx} \tag{1}$$

where $I(x)$ is an image which has a real-value data, X is the field of view of image, c is a coefficient that is directly related to scaling parameter of FREBAS transform $D=\pi/(cN\Delta x^2)$, and m is an index number of higher components of images that corresponds to the higher frequency components of images. In case, FREBAS transformed domain is limited spatially by width x_w, the effect is described as the multiplication by rectangular function $\mathrm{rect}(x/x_w)$ to i_m. Inversely FREBAS transformed image I_m is obtained by the convolution integral of Eq. (1) and the function, which is the inverse of filter function $g(x)$ as shown in Eq.(2). Real part of a reproduced image is obtained by the sum of the complex conjugate function as shown in Eq.(3).

$$I_m = \left(\mathrm{rect}\left(\frac{x}{x_w}\right)\cdot\left[I(x)e^{-jcx^2} * g(x)\right]\right) * \bar{g}(x) \tag{2}$$

$$\mathrm{Re}\,[I_m] = \frac{1}{2}\left(\mathrm{rect}\left(\frac{x}{x_w}\right)\cdot\left[I(x)e^{-jcx^2} * g(x)\right]\right) * \bar{g}(x) + \frac{1}{2}\left(\mathrm{rect}\left(\frac{x}{x_w}\right)\cdot\left[I(x)e^{jcx^2} * \bar{g}(x)\right]\right) * g(x) \tag{3}$$

Finally, the following image is obtained by once again applying the FREBAS transform to Eq. (3):

$$i'_m = \mathrm{Re}[I_m]e^{-jcx^2} * g(x) = \frac{1}{2}\left(\mathrm{rect}\left(\frac{x}{x_w}\right)\cdot\left[I(x)e^{-jcx^2} * g(x)\right]\right) + \frac{1}{2}\left(\mathrm{rect}\left(\frac{x}{x_w}\right)\cdot\left[I(x)e^{-jcx^2} * \bar{g}(x)\right]\right) * g(x) * g(x) \tag{4}$$

The first term of Eq. (4) is just half of the original band-limited function itself. The second term is a convolution integral of band-limited function with $g(x)$, therefore the distribution of signal exists beyond the restricted band $\mathrm{rect}(x/x_w)$. This is the principle of signal extrapolation in the FREBAS transformed domain.

Method: Figure 1 shows a schematic diagram of the image magnifying procedure using FREBAS transform. FREBAS transform is applied to the original image (NxM) using scaling parameter D (Fig. 1(a)), and the signal is expanded to be 2Nx2M by the zero padding outside the signal as shown in Fig. 1(c). The inverse FREBAS transform is then applied using scaling parameter 2D to obtain a magnified image. Since the image is real-valued data, the real part of the magnified image (d) is FREBAS transformed again, and central NxM region is replaced by the original FREBAS signal (b) as true data. The iterative algorithm from (c) to (d) can extrapolate signal in the FREBAS domain and hence, the magnified image (d) has higher resolution than the magnified image with simple interpolation by sinc function.

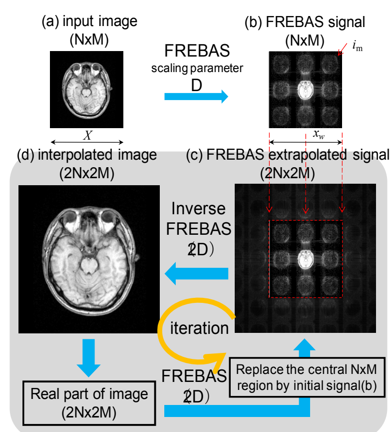

Fig.1 An algorithm for image interpolation using FREBAS transform

Results and Discussion: Figure 2 shows the demonstration of proposed method. The central 128x128 region in the FREBAS 256x256 signal was remained and outside the region was zero-padded. Proposed method was applied to this band-limited signal. Comparison of extrapolated signal with true signal show that extrapolated signal is close to true signal in the region adjacent to the central band-width. Improvement of resolution is shown in Fig.3, where resolution improvement is evaluated by the sharpness of point spread function. The region where image is strongly sharpened depends on whether the number of scaling parameter D is odd or even. Figure 4 shows the results using MR and test chart images model (D = 3). In Fig. 4, image (a) and (c), are the interpolated images by zero-filling in the Fourier transformed domain. Image (b) and (d), obtained by applying the proposed method, become clearer and much more sharpened than that of image (a) and (c).

Conclusion: Interpolation of images with super-resolution effect using the combination of FREBAS transform and Gerchberg-Papoulis algorithm is presented. It was confirmed that signal can be extrapolated in the FREBAS transformed space and therefore, the magnified images were much more sharpened with super-resolution effects compared to the images by zero-filling in the Fourier transformed space.

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[3] Ito S, et al, IEEE ICIP 2003, Barcelona, Spain, Map8.7, 2003.

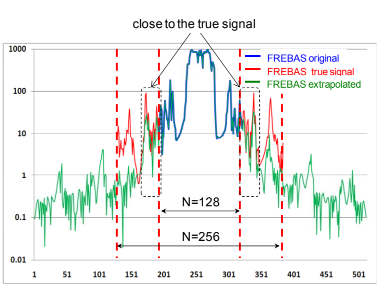

Fig.2 Comparison of extrapolated signal with true signal

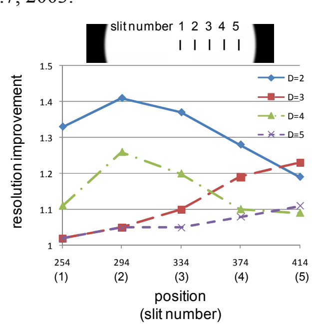

Fig.3 Result of resolution improvements

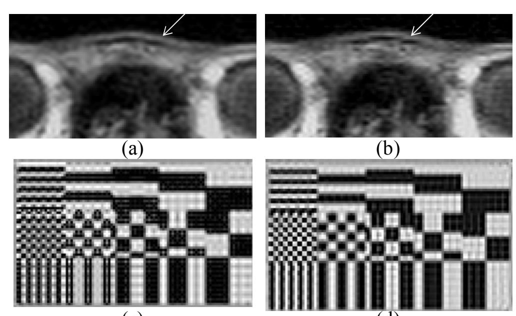

Fig.4 Interpolated images: (a),(c) interpolation by sinc function, (b),(d) FREBAS interpolation (D=3)