# Estimation of Superresolution Performance

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**Introduction:** Recently, it has been proposed that sensitivity modulations of the receiver coils might lead to an improved image resolution (*superresolution*) by alteration of the standard reconstruction method [1]. Simulation results and initial experimental evidence [2] have shown that low-resolution imaging modalities like magnetic resonance spectroscopic imaging potentially profit more than high-resolution applications. So far, it ha been argued only qualitatively that the sensitivity modulation over nominal voxels could be exploited to improve image resolution. The aim of this contribution is to provide a method, which is suitable for estimating the possible effect of the used rf-receiver array onto the superresolution performance.

**Theory**: Image resolution can be investigated comprehensively by analyzing the point spread function (PSF). For the strong reconstruction approach [3] or approximations thereof [1], the PSF is the Hermitian of the spatial response function (SRF), i.e. it has similar properties and it is sufficient to analyze either of them. Whereas the PSF describes to which voxels a single spin contributes, the SRF describes the signal origins contributing to the image voxel of interest. For Fourier encoding, SRF and PSF are a sinc-function. In the Fourier-domain representation, the SRF is a superposition of the (Fourier-domain-)coil sensitivities shifted by the different acquired k-space positions. The weighting coefficients depend on the reconstruction method:

$$\widehat{SRF}_{\rho}(\vec{k}) = \sum_{\alpha,\kappa} F_{\rho,(\alpha,\kappa)}\,\widehat{C}_{\alpha}(\vec{k}-\vec{k}_{\kappa}) \qquad (1)$$

A wider k-space support of the SRF corresponds to a narrower PSF with improved resolution. For Fourier encoding with a homogeneous coil the achievable width is restricted to the acquired k-space grid (Fig. 1, left). In parallel imaging with non-homogeneous coil sensitivities, however, the finite width of coil sensitivities in k-space allows an extension of the k-space support of the SRF (Fig. 1e,f) independent of the k-space grid – in absolute numbers. However, in relative numbers, the extrapolation zone becomes more relevant for low resolution images. For an effect of 1% for 256 k-space lines, the effect would be 16 % for 16 lines.

**Methods:** 1D and 2D PSF simulations were performed using MatLab (The Mathworks, USA). In contrast to coil-by-coil reconstruction algorithms, the strong reconstruction approach [3] (in this abstract we refer to it as "superresolution reconstruction", even though similar reconstruction methods might be used) with the reconstruction matrix $F = E^H C^+$ was applied. $E$ is the encoding matrix. The correlation matrix of the encoding functions $C$ was approximated by $EE^H$ . Thikonov regularization was used for the calculation of the pseudo-inverse. 8 or 16 k-space lines/rows were simulated and a fine reconstruction grid was chosen (1024 for 1D and 256x256 for 2D). The model was verified using idealized coil sensitivities with finite box-shaped support in k-space. The excitation of a small voxel was simulated by calculating the signal for 100 spins in the corresponding voxel. Also, real-world sensitivity maps of an rf-head coil were measured on a 3T Tim Trio (Siemens, Germany). The sensitivity maps were interpolated onto a fine grid during the reconstruction procedure. Experimental data were acquired using a 2D CSI sequence. Only a fraction of a nominal voxel was excited to mimic a PSF measurement.

**Results:** Fig. 2 shows 1D simulation results. In combination with Fig. 1 the plots clearly show that resolution (defined here as the distance between zero-crossings in the PSF) increases with increasing SRF k-space support. Moreover, there even is a decrease in side-lobe intensity corresponding to a reduced Gibb's ringing artifact. For the simulated finite-support sensitivity profiles the resolution is 50% higher than for Fourier imaging (Fig. 2b) exactly corresponding to a 50% increase in k-space support (Fig. 1e). Using the measured sensitivity profiles, the resolution gain is 32 %. In Fig. 3 and Fig. 4 measurement results are shown, which match very well to simulated data. For 8x8 phase encoding steps the resolution gain was 32.7 %, for 16x16 about half the gain (16.2 %). However, it is to be noted that the resolution improvement depends on the Thikonov regularization parameter (Fig. 5).

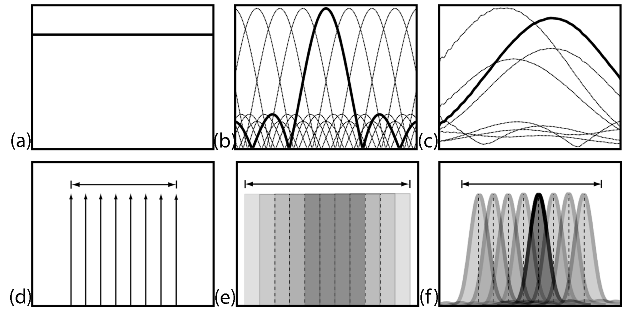



**Fig. 1:** (1D) Sensitivity maps (top) and corresponding SRF k-space support (bottom) for 8 gradient encoding steps. Left: Homogeneous profile. Middle: 8 simulated profiles with finite support in k-space. Right: Central profiles of measured sensitivity maps. The width of k-space support for the resulting SRF increases with higher variations of the sensitivity profiles.

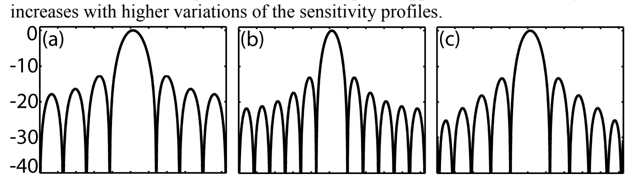
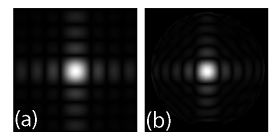



**Fig. 2:** Superresolution-PSF ordered in the same way as in Fig. 1. Depicted are log-plots in decibel. The width of the main lobe and numbers of side lobes scale according to the increase in spatial support of the corresponding SRF.

**Fig. 3:** PSF for 2D conventional coil-by-coil reconstruction (a) and superresolution reconstruction (b). 8x8 k-space points were measured. There is some resolution improvement visible in (b).



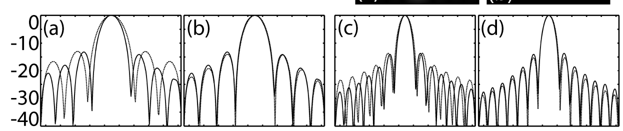




**Fig. 4:** PSF results with 8x8 (a,b) resp. 16x16 (c,d) phase encoding steps. Shown is a 1D-profile through the center of the 2D-PSF. (a,c) show measurement results. Standard coil-by-coil reconstruction (broad PSF) is compared to superresolution reconstruction (narrow PSF). (b,d): The measurement results are compared to simulation results showing very good reproducibility. The sensitivity effect on resolution is higher for smaller k-space grids.

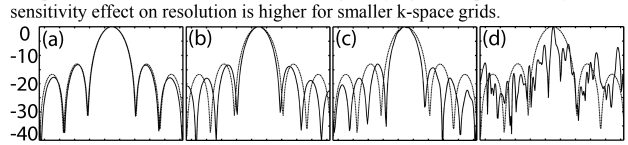



**Fig. 5:** Profiles of the 2D-PSF superresolution reconstruction with different Thikonov regularization parameter (narrow PSF) compared to coil-by-coil reconstruction (broad PSF). The Thikonov parameter is chosen such that the ratio of highest to lowest eigenvalue changes from left to right: 2, 100, $10^4$, $10^6$. With high Thikonov parameter the correlations are underestimated; almost no resolution improvement occurs. For low parameters the correlations are adequately accounted for; resolution is improved at the expense of noise artifacts.

**Discussion:** For simulated sensitivity maps, box-shaped in k-space, the model predicts the correct resolution gain. In contrast to the simulations, real-world sensitivity data have no clearly defined cut-off frequency in k-space. It can be controlled indirectly by Thikonov regularization resulting in a trade-off between resolution and SNR (Fig. 5). In spite of this flexibility, good estimations of the superresolution performance are found by analyzing the k-space extent of the sensitivity maps (cp. Fig. 1d-f). From visual inspection of the k-space support in Fig. 1f it is to be expected that the reconstruction should perform well for a resolution gain of at least 25 %. The reconstruction is actually well-behaved even for a resolution increase of 32 % (Fig. 4a).

The expected inverse relationship of superresolution performance and extent of the gradient-acquired k-space could be verified experimentally (cp. Fig. 4a,c). On current 3T systems a significant superresolution effect can only be expected for low resolution applications. However, at field strengths over 3T rf-sensitivity modulations are higher and the effectiveness of superresolution should be enhanced. Particularly interesting are also encoding schemes with nonlinear encoding fields (SEMs), which generate spatially varying resolution [4,5]. Lack of resolution due to the SEM-system could be compensated for by incorporating the sensitivity modulations into the reconstruction.

The strength of the presented method is to provide easily accessible information about the global encoding capabilities of the used receiver coil array. Whereas in GRAPPA [6] the finite width in k-space of the receive coil sensitivities is used to fill space between measured k-space lines, in Superresolution, it is used to extrapolate the k-space beyond the borders given by gradient encoding. This is achieved by also considering correlations in the encoding functions, which is not the case in standard reconstruction algorithms.

**Acknowledgements:** This work is part of the INUMAC project supported by the German Federal Ministry of Education and Research, grant #13N9208.

**References:** [1] Sànchez-Gonzales et al., MRM 55, 287–295, 2006; [2] Otazo et al., NeuroImage 47, 220–230, 2009; [3] Prüssmann et al., MRM 42, 952–962, 1999; [4] Hennig et al., MAGMA 21(1-2):5–14, 2008; [5] Schultz et al., Proc. ISMRM, 786, 2008; [6] Griswold et al., MRM 47, 1202–1210, 2002.