System Dynamics Estimation for Kalman Filtering With Radial Acquisition

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INTRODUCTION
A Kalman filter provides causal operation, which is desirable in several rapid imaging applications. A successful real-time implementation of a Kalman filter has recently been developed used with spiral trajectories for applications in dynamic cardiac MRI and has been shown to outperform an existing sliding window method [1,2]., However there are different ways to estimate parameters of the filter. In this study, a method for estimating dynamics of the imaging application using a causal adaptation of a temporal filtering method based on a Density Compensation Function (DCF or tornado filter [3]) is presented. The method is demonstrated on a cardiac phantom for with a significantly undersampled radial acquisition. Radial acquisition may also provide other benefits, as their shorter data acquisition intervals per TR limit off-resonance blurring.

THEORY
As in [1], we assume that the image at any given time frame is the image in the previous time frame plus some small change *u(t)*. True images are considered system state at any given time point *s(t)*. Observable k-space data *x(t)* is related to the image through Fourier transform (*F*) and inverse gridding *G(t)* and deapodization Γ^{-1} linear operations. The observation noise *ω(t)* is thermal additive white Gaussian noise. The model is summarized as: $s_{t+1} = s_t + u_t; x_t = G_t F \Gamma^{-1} s_t + \omega_t$ and the reconstruction is formulated as in [1]:

$$H = GF\Gamma^{-1}; Z = H^H H$$
$$P \leftarrow P + Q; P \leftarrow P \div (1 + P \times Z)$$
$$\hat{s} \leftarrow \hat{s} + P \times [H^H((x - H\hat{s})/\sigma^2)]$$

where × and ÷ are element-by-element operations, $\hat{s}$ is our estimation of true image at every time frame. Matrix *P* shows estimation error for every pixel and σ^2 is acquisition noise. The success of the filtering is dependant on the knowledge of the covariance matrix *Q≡cov(u(t))* or *motion map* [2].

Without using training scans, there are two choices for the motion map: 1) using low spatial resolution data with high temporal resolution or 2) using images with high spatial resolution that is temporally blurred. A spiral sliding window algorithm is used to generate the motion map in [2]. Our approach for highly variable rate sampling strategies uses a DCF-based temporal (tornado) filter which narrows at the center of k-space and has proven to outperform sliding window [3]. If an asymmetric temporal filter uses only over frames acquired prior to each frame, the tornado filter can be causal too as shown in Figure 1. Choosing an asymmetric tornado filter is compatible with real-time operation and generates a motion map (*Q*) with less temporal blurring.

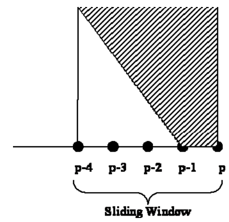
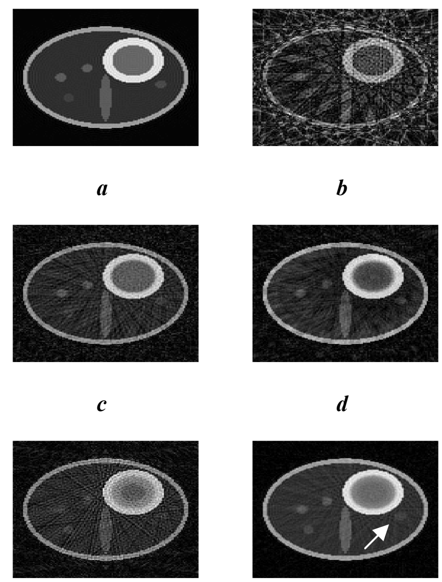


Figure1. *Causal Tornado Filter weighting vs. causal Sliding Window*

METHODS
The phantom simulates moving tissue in dynamic cardiac imaging and signal intensity change in the myocardium and surrounding vessels. A 2-D 128×128 digital phantom with 40 different time frames was used. The phantom had a large ellipse that changes size and intensity to simulate the myocardium and 5 smaller simulated vessels [4]. Images were sampled on radial trajectories with a bit reversal interleaving pattern to acquire unique projections in every time frame. The database is heavily undersampled with only 10 projections acquired every time frame. A sliding window and a tornado filter regular reconstructions have also been used for comparison purposes. To create a fair comparison, both filters have the same maximum window aperture.

RESULTS AND DISCUSSION
Figure 2 (a) shows the fully sampled true phantom at a given time frame; (b) is the reconstruction using only data from the heavily undersampled time frame; (c) is the result using the conventional sliding window reconstruction with five time frames and (d) is the result of a causal tornado filter. Figure 2(e,f) are Kalman reconstructions with our suggested implementation with (e) using a five frame conventional sliding window images to update motion map and (f) using the suggested causal tornado filter to estimate system dynamics. It is seen in Figure 2f that undersampling artifacts are reduced and edge detail is more consistent with the truth. The arrow shows a very small vessel with low intensity which is lost if our motion map is blurry (Figure 2e) but can be depicted now with the new motion map selection (Figure 2f). Comparing (d) and (f) also shows that we can likely improve DCF-based reconstructions by using Kalman adaptive filtering.



e ***f***

Figure2.-*(a) Fully sampled phantom (b) single frame reconstruction without view sharing (c) Sliding window reconstruction (d) Tornado filter reconstruction (e) Kalman filter with sliding window motion map (f) Kalman filter with Tornado motion map- Arrow shows a small feature visible in f*

CONCLUSION
A Kalman filter has been simulated using radial undersampling pattern with two different selections for Kalman motion map. The motion map tuned to the variable sampling radial trajectory produces an improved reconstruction over using a sliding window to estimate the motion map. Using adaptive filtering also improved causal tornado filtering. The results show great potentials of the filter for perfusion studies and dynamic cardiac MRI. The total process is causal and have manageable computational load which makes it appealing for real time applications.

REFRENCES
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