Maxwell's Equation Tailored Reverse Method of Obtaining Coil Sensitivity for Parallel MRI

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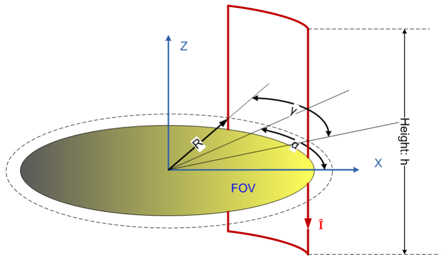
Introduction: A new method is proposed to obtain noise-free RF coil sensitivity maps. This is highly desirable, considering the fact that the sensitivity encoding (SENSE)[1] method imposes ultimate dependence of successful full FOV image reconstruction on the correct sensitivity map of each individual coil. The proposed method differs from traditional methods in that, instead of refining the measured sensitivity maps by means of numerical approximation and/or extrapolation, it is based on physics of electromagnetics, parameterization and optimization algorithms.



Figure–1 Coil Geometry and Model Descriptors

Methods: Noise is inevitably found in any MR images and consequently in measured sensitivity maps. For the purpose of minimizing noise from coil sensitivity profiles, the proposed method is based on the concept of our previous work on exposure evaluations by reverse-engineering of gradient coils[2] – in analogy, the geometry of a typical RF coil can be inversely determined from the measured magnetic field information: the RF coil may be represented using a few model descriptors; the principle of reciprocity enables us to reproduce the measured B1 field using Maxwell's equation (Biot – Savart Law at low frequencies) with resonant current flowing in the coil element; with appropriate parameterization, an optimization algorithm can be used to attain the values of descriptors numerically, and consequently to determine the RF coil geometry. Once the coil geometry is determined, one can easily evaluate the coil sensitivity maps with controllable noise.

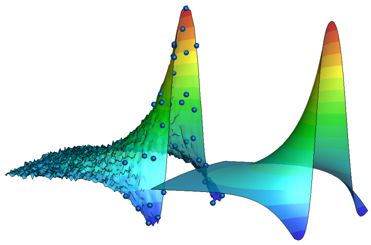


Figure-2 Noise-Free (right) and Noise-Added Sensitivity Profile (left) with Sampling Points Representing Sensitivity Characteristics (blue dots)

An element of a typical RF coil array is modelled using a few descriptors, as shown in Figure-1, where γ represents the opening of the coil element, α represents the shift with respect to the 0° of the cylindrical coordinate system, Ī represents the resonant current. To evaluate the performance of the proposed method quantitatively, a noise-free sensitivity profile is calculated as depicted in Figure-2 (right) employing electromagnetic simulation software FEKO (EMSS - SA). Model parameters: height h=90mm, radius R=37.6mm, α=0° and γ=35°. The coil element is tuned to resonate at 64MHz and the magnetic field in X-Y plane is studied at Z=0. A various amount of noise with amplitude ranging from -70dB to -5dB with respect to signal is added to the, otherwise noise-free, FEKO simulation results. Signal amplitude is derived from the following procedure (real and imaginary parts of the simulated profile are examined separately): a region of relatively high amplitude is chosen, the average of which is taken as signal amplitude. Noise is generated accordingly, assuming zero mean Gaussian distribution in both real and imaginary channels. Figure-2 (left) plots the noise-added B1 field. 40 points with a vector of coordinates V_r are selected to describe the characteristics of the profile as shown by the blue dots in Figure-2 (left). The optimization process is then utilized to minimize the difference between sampled and calculated points:

$$\min_{\alpha,\gamma,I}\left[(H_v - H_1)\times(H_v - H_1)^H\right]$$

	optimization					Polynomial Fit			
	optimization result					order = 2		order = 4	
Noise level	alpha (°)	gamma (°)	current	RMSD	PSNR(dB)	RMSD	PSNR(dB)	RMSD	PSNR(dB)
-70dB	-0.0159	35.0086	-3.9657	0.16	52.827	1.545	33.12	1.545	33.118
-60dB	-0.0147	34.9991	-3.9661	0.162	52.711	1.547	33.109	1.549	33.099
-50dB	-0.0145	35.0057	-3.9660	0.168	52.394	1.553	33.077	1.559	33.042
-40dB	-0.0297	34.9985	-3.9648	0.17	52.28	1.571	32.973	1.59	32.868
-30dB	-0.0678	34.9414	-3.9792	0.178	51.881	1.631	32.65	1.688	32.351
-20dB	-0.0085	34.6655	-4.0129	0.35	46.029	1.802	31.782	1.966	31.029
-10dB	0.2306	33.6031	-4.0861	0.494	43.026	2.262	29.807	2.652	28.428
-5dB	-0.916	33.9848	-4.0488	0.74	39.515	2.699	28.274	3.264	26.622

Table-1 Comparison of RMSD and PSNR between Proposed Method and Polynomial Fit

where H_v is calculated magnetic field using Biot – Savart law at sampled coordinates V_r; H_1 represents the complex-valued field strength in the noise-added profile at V_r. The optimized profile H_{opt} is obtained by substitution of optimized values of descriptors into the mathematical model. Various orders (from 2 to 8, 6 to 8 is not shown here) of polynomial fitting are performed to acquire noise-reduced profiles H_{poly}. The root mean square deviation (RMSD) and peak signal to noise ratio (PSNR) of ($H_{opt} - H_1$) and ($H_{poly} - H_1$) are calculated at various noise levels. H_{opt} and H_{poly} are then used to reconstruct a phantom image using SENSE method with reduction factor of three (R=3), the results of which are compared.

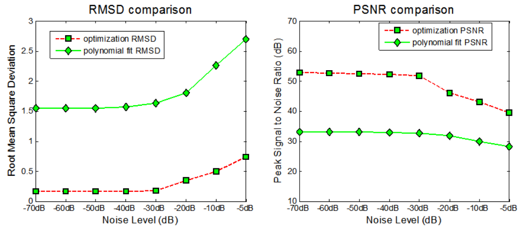


Figure-3 RMSD (left) and PSNR (right) of the Difference between Noise-Free Sensitivity Map and Sensitivity Map Derived by Proposed Method (Red), or Polynomial Fitted (Green)

Results and discussion: The results shown in Table-1 and Figure-3 demonstrate that, for a noise level ranging from -70dB to –5dB, the sensitivity profiles produced by the proposed method have RMSD more than one order of magnitude smaller than that of polynomial fit; and the PSNR of proposed method is more than 10dB larger than that of polynomial fit. It becomes apparent the SNR advantage of incorporating the proposed method into parallel NMR practice, where SNR is typically ranged from -35dB to -15dB measured by aforementioned method. SENSE reconstruction is simulated with a level of -25dB noise is added to the sensitivity map. Figure-4 illustrates the benefit, in terms of reconstruction errors, of applying the proposed method to SENSE reconstruction compared to the application of the traditional polynomial fitting method.

Conclusion: The simulation results suggest that the proposed algorithm can precisely predict the coil array geometry and resonant current, which in turn can be used to construct the coil sensitivity maps. This theoretical study indicates that the Maxwell's equation tailored inverse method brings about substantial SNR advantages in sensitivity map formation compared with the traditional polynomial fitting method, and is feasible for parallel MR image reconstruction.

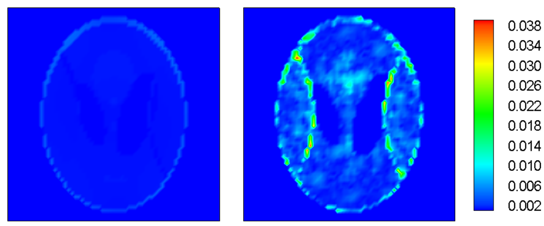


Figure-4 Difference between Original Phantom Image and SENSE-Reconstructed Image by Proposed Method (left) and Polynomial Fit (right)

[1]Pruessmann, K. P., et al. Magnetic Resonance in Medicine, 1999. 42(5): p. 952-962.
[2]Liu, F., et al. Conc. in Mag. Reson. Part B: Mag. Reson. Eng., 2009. 35B(1): p. 32-43.