Conjugate Gradient PINOT Reconstruction with a Fast Initial Estimate

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INTRODUCTION: PINOT[1] (Parallel Imaging and NOquist in Tandem), combining SPACE-RIP for parallel imaging and Noquist for spatiotemporal redundancy, favorably preserves edge detail at a cost of increased SNR. PINOT reconstruction involves the direct inversion of a large matrix for each read-out coordinate, combining all time frames and coils. A solution for alleviating the high computational cost of this direct matrix inversion is the use of iterative algorithms, such as the conjugate-gradient (CG) method[2]. CG-PINOT is investigated in this abstract. Additional significant time savings is achieved by providing a favorable initial estimate[3]. We call this approach CG-initialized PINOT (CGi-PINOT).

METHOD: PINOT reconstructs the image f, constrained by spatiotemporal and coil sensitivity priors, from reduced data F by direct-inversion of the forward model $F = M_{PINOT} f$. The size of M_{PINOT} (eq. (1)) is $(T\,C\,N_s) \times (TD+S)$. N_s is the number of sampled phase encodings out of N for each time point, T is the number of time points, and C is the number of coils. S and D are the sizes of the static and dynamic portions of the field of view, so $S+D=N$. The CG convergence speed depends on the initial estimate f_0. By default, $f_0 = 0$. Here, we use an estimate of f_0, which is very close to f, taking advantage of special structure of sparse matrix M_{PINOT}. We approximate the pseudo-inverse of M_{PINOT} as Eq. (2). Due to the block structure of M_{PINOT} and this pseudo-inverse, we are able to separate the problem by frame index t and then deconstruct the problem into a series of pseudo-inversions for each frame as shown in Eq. (3). Each of these inversions can be solved at very low computational cost. Since this is not the least-squares solution, we lose some noise resistance relative to solving the full matrix. This result is too noisy to use as the final reconstruction, but provides a good initial estimate f_0, and allows the CG-PINOT to be solved with fewer iterations.

$$M_{PINOT} = \begin{bmatrix} S_1^{N_s\times S} & D_1^{N_s\times D} & 0 & \cdots & 0 \\ S_2^{N_s\times S} & 0 & D_2^{N_s\times D} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ S_T^{N_s\times S} & 0 & 0 & \cdots & S_T^{N_s\times D} \end{bmatrix} \quad (1)$$

$$M_{PINOT}^{\dagger} = \begin{bmatrix} X_1^{S\times N_s} & X_2^{S\times N_s} & \cdots & X_T^{S\times N_s} \\ Y_1^{D\times N_s} & 0 & \cdots & 0 \\ 0 & Y_2^{D\times N_s} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Y_T^{D\times N_s} \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} TX_t \\ Y_t \end{bmatrix} \begin{bmatrix} S_t & D_t \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (3)$$

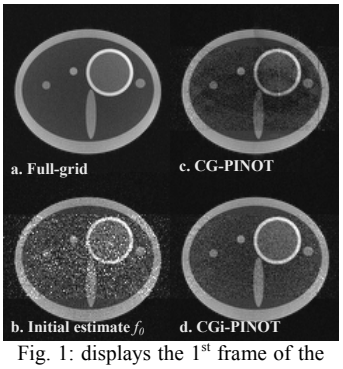

Fig. 1: displays the 1st frame of the phantom images, showing the full-grid reconstruction (a), initial estimate f_0 (b), CG-PINOT (c) and CGi-PINOT (d) at 10 iterations.

EXPERIMENTS: We compare CGi-PINOT, CG-PINOT (0 as initial estimate) and direct matrix inversion PINOT together in terms of reconstruction time, convergence speed and number of iterations, on both simulated and in-vivo scans. The number of iterations and tolerance were chosen to achieve comparable image quality to direct inversion PINOT. The computer simulated phantom data has image size 120 × 120, T=15, C=4 for reduction factor R_{PI}=3.75, combining SPACE-RIP (R_p=2) with Noquist (R_{nq}=1.875). A short axis ciné MRI scan was acquired on a 1.5 Tesla GE Signa TwinSpeed scanner with image size 224 × 256, T=16, and C=8. (TE=2.0 ms, TR=4.1 ms, flip angle=45°, FOV=35×35 cm, slice thickness=12 mm). R_{PI} is 3.76 with R_p=2 and R_{nq}=1.88. All reconstructions were computed with MATLAB on a Quad Core Xeon 2.66GHz computer with 16GB of RAM.

Table 1. Time (min) and iteration numbers needed for direct inversion PINOT, CG-PINOT and CGi-PINOT.

	Phantom		Cine MRI	
	Time	Iter.	Time	Iter.
Direct Inver.	58.01	N/A	1595.5	N/A
CG-PINOT	6.38	20	70.35	40
CGi-PINOT	4.57	15	67.23	30

RESULTS: It takes about 10s to calculate f_0 for the phantom data, which is negligible compared to the full reconstruction time. Table 1 shows for the same image qualities, CG/CGi-PINOT are much faster than the direct inversion PINOT, while CGi-PINOT converges faster than uninitialized CG-PINOT. Fig. 1 shows a comparison of CG-PINOT and CGi-PINOT after 10 iterations for phantom data. CGi-PINOT almost converges while CG-PINOT still has artifacts. The initial estimate f_0 (Fig. 1b) reconstructs quickly but has significant noise. Ciné MRI reconstruction (Fig. 2) shows similar characteristics but CGi-PINOT converges at 30 iterations, while CG-PINOT converges at 40 iterations. We also tested CG-PINOT and CGi-PINOT running until the CG method has fully converged, typically 100~300 iterations, which take about 152.34 min., still 10 times faster than direct inversion PINOT. The results (Fig 2.g) were equal for CG-PINOT and CGi-PINOT and have slightly lower RMS errors than the images shown in Fig. 2.

DISCUSSION AND CONCLUSIONS: The CG method converges typically in three phases, an initial phase of rapid convergence but short duration, which depends essentially only on the initial error[3]. Both simulated and in vivo studies show that CGi-PINOT with initial estimate f_0 converges faster and provides excellent reconstructed image quality with less time. This is due to the initial convergence phase of CG method. Furthermore, as temporal frames and/or the image size increase, the size of M_{PINOT} will increase proportionally, causing the calculation time advantage of CGi-PINOT to increase dramatically. However as the number of iterations increases beyond the initial convergence phase, the convergence rate no longer depends heavily on initial error, and the speed advantage of CGi-PINOT disappears. The results then have higher accuracy than direct inversion because the CG method is more resistant to numerical precision problems. Even at complete convergence, CGi-PINOT requires a full order of magnitude less computation time than direct inversion.

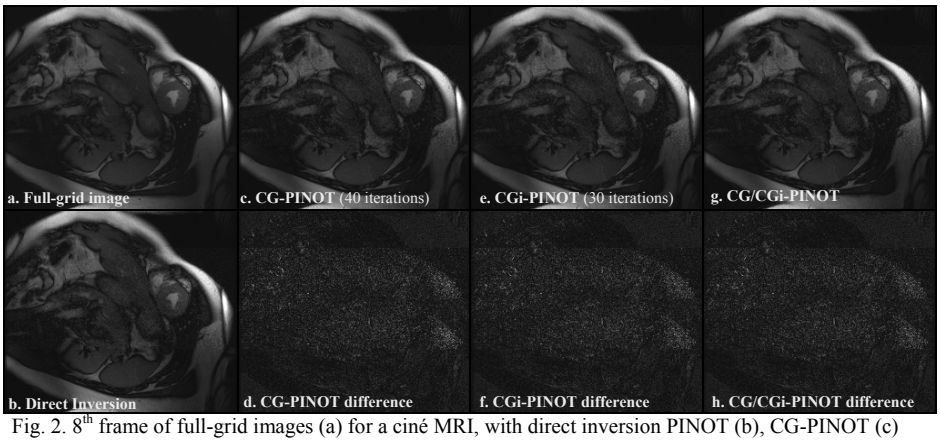

Fig. 2. 8th frame of full-grid images (a) for a ciné MRI, with direct inversion PINOT (b), CG-PINOT (c) and CGi-PINOT (e) at 40 and 30 iterations respectively. The fully converged CG/CGi-PINOT are shown in (g). The bottom row (d), (f) and (h) shows the corresponding difference images (intensity enlarged 5 times to show details) from the corresponding top. All images are under the same scale.

REFERENCES

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