Towards a geometry factor for projection imaging with non-linear gradient fields

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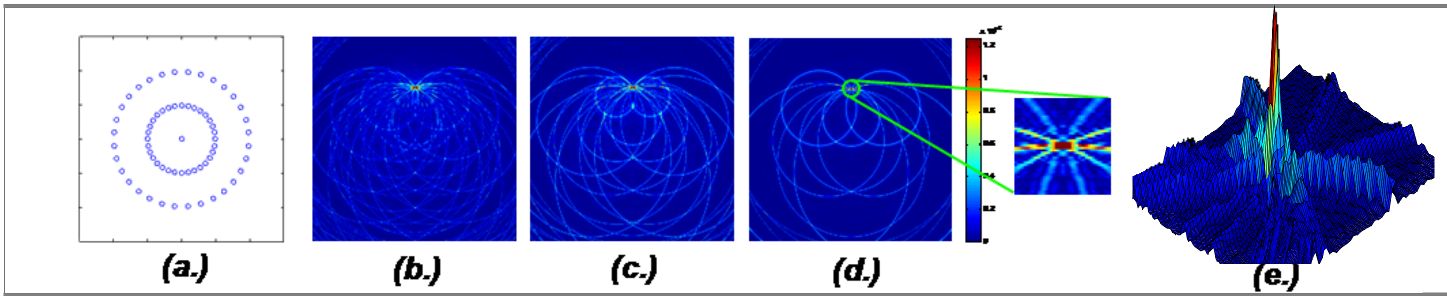
INTRODUCTION: O-Space parallel imaging combines a Z2 gradient field with conventional radial k-space trajectories to perform highly-efficient parallel imaging [1]. Because the Z2 field varies as $r^2 = x^2+y^2$ in the axial plane, it provides spatial encoding that is complementary to that obtained from a circumferential surface coil array. However, non-linear projection encoding invites the exploration of a great multitude of trajectories, including differently-weighted combinations of X, Y, Z2 producing projections with different center placements (CPs). Time-varying readout pulses such as those used in spiral acquisitions present yet another degree of freedom for non-linear encoding. At present it is unclear how to best traverse this large space of potential encoding functions. A good first step is to develop a metric for evaluating the performance of arbitrary sets of surface coils, gradients, and pulse shapes. This work presents a protocol for evaluating the performance of these trajectories with a given surface coil array via and attempts to formulate an O-Space equivalent to the conventional g-factor.

In conventional parallel imaging, the spatially-varying noise amplification of SENSE [2] and GRAPPA [3] reconstructions is described by the geometry factor, or g-factor. Maps of the g-factor illustrate the degree of noise amplification throughout the image while assuming that resolution remains invariant. A direct extension of Cartesian SENSE into the domain of non-linear encoding gradients has been proposed in the form PatLoc imaging [4]. PatLoc uses pairs of orthogonal, multipolar gradients, one of which is used for readout and the other for phase encoding. Because the shapes of the gradients do not change in between readouts, the Jacobian of the linear-to-multipolar transformation can be used to calculate voxel size throughout the FOV. The PatLoc g-factor is thus simply the SENSE g-factor weighted by the Jacobian. By contrast, in O-Space imaging, the spatially-varying resolution is a complicated function of the CPs selected for each projection.

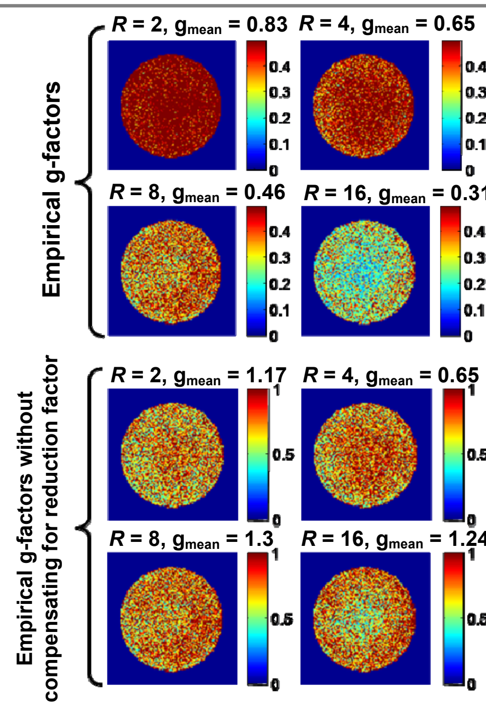


(a.) Typical center placement (CP) scheme for *R*=2. Evenly-spaced subsets of these centers are used for *R*=4, *R*=8, and *R*=16 O-Space encoding. Point spread function estimates are plotted for a representative point with *R*=4 ***(b.)***, *R*=8, ***(c.)***, and *R*=16 ***(d.-e.)***. (effect of coil profiles is omitted)

$$g_\rho \equiv \frac{SNR_\rho^{full}}{SNR_\rho^{red}\sqrt{R}} \cong \frac{\mu_\rho^{full}/\sigma_\rho^{full}}{\sqrt{R}\,\mu_\rho^{full}/\sigma_\rho^{full}}$$

$$s_{m,l}(t) = \iint \rho(x,y)C_l(x,y)e^{-j2\pi G_{Z2}\frac{1}{2}\left((x-x_m)^2+(y-y_m)^2\right)t}dxdy = A_{m,l,t}\rho$$

In O-Space imaging, iterative methods such as the Kaczmarz projection algorithm [5] are used to find a solution to the entire encoding problem, $s = A\rho$, spanning all readout samples, coils, and applied gradient shapes. The condition number of the encoding matrix is the ultimate metric of encoding performance, providing information both about noise amplification and resolution. However, the encoding matrix is typically too large (~$2^{16}\times2^{14}$) for the condition number to be obtained via singular value decomposition. In this work we explore the possibility of evaluating O-Space reconstructions using the empirical g-factor obtained through Monte Carlo simulation. **METHODS:** Simulations are performed using an 8-channel coil array with a 128×128 numerical phantom consisting of ones everywhere within a circular mask. Reconstructions with acceleration factors of R={1,2,4,8,16} are simulated, corresponding to 128, 64, 32, 16, and 8 echoes, respectively. Noise with standard deviation equal to 0.15 is added. Thirty Monte Carlo replicates are reconstructed and the voxel-wise mean and standard deviation are computed for the "fully-sampled" and undersampled reconstructions. **DISCUSSION:** The empirical g-factor actually *decreases* with the acceleration factor, becoming less than unity for $R > 1$. Clearly, it is an unsuitable metric of assessing performance in non-linear projection imaging. The reason for this anomaly is that the "fully-encoded" reconstructed passes more noise to the image by virtue of its superior resolution. By contrast, the R=16 reconstruction shows considerably less noise in the FOV center where the voxel size is large. Since signal is proportional to voxel area, the g-factor maps should be divided by the voxel size at each point. Voxel size can be estimated using the second-order moment of the PSF about each voxel. The PSF can be obtained by simply encoding and reconstructing from a delta-source image, but this is too computationally-demanding to perform for multiple rival encoding functions. A computationally tractable method of obtaining the PSF at each voxel is therefore used and the resulting solution accounts for both the noise and resolution in O-Space accelerated acquisitions.



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