Tailored 3D Random Sampling Patterns for Nonlinear Parallel Imaging

F. Knoll[1], C. Clason[2], and R. Stollberger[1]

[1]Institute of Medical Engineering, Graz University of Technology, Graz, Austria, [2]Institute for Mathematics and Scientific Computing, University of Graz, Graz, Austria

Introduction: Within the framework of compressed sensing, Lustig et al. [1] proposed the idea of randomized 3D Cartesian subsampling. These randomized sampling patterns can be generated by using Monte-Carlo simulations. With this approach, a probability density function (pdf) is constructed, and sampling pattern indices are drawn randomly according to that density. This leaves the problem of designing a pdf that will generate a suitable sampling pattern, and especially of determining the correct ratio of low to high frequency sample points. A common approach uses a density function that is proportional to a power of the distance from the origin in k-space. However, such strategies require optimization of one or even more design parameters which have significant influence on the reconstruction quality. The goal of this work is to show that it is possible to construct an adapted random sampling pattern from a pdf that is generated by using measured k-space data as a reference, which automatically ensures an appropriate distribution of sample points. Data sets acquired using such a pattern can be reconstructed using regularized nonlinear inversion for parallel imaging [2]. This allows the use of very high acceleration factors while still yielding images with excellent image quality.

Methods: Subsampling experiments were performed to evaluate the proposed method. Fully sampled T_2 weighted scans of the brain (2D TSE, TR/TE=5260/80ms, turbo factor 5, matrix 256x256, 40 slices, slice thickness 3mm, in plane resolution 0.86mm x 0.86mm) were acquired on a clinical 3T scanner (Siemens Magnetom TIM Trio, Erlangen, Germany), using an 8 channel receive coil. Written informed consent was obtained from all subjects prior to the examinations. Raw data sets were exported and subsampled, to simulate an accelerated acquisition. Random sampling patterns were constructed with the use of pdfs that were generated with data sets from different volunteers, which were acquired several weeks prior to these examinations. Subsampling with reduction factors R=4, R=10 and R=18 was performed, and the proposed method was compared to conventional cartesian subsampling (2x2, 3x3, 4x5 with 24 additional reference lines at the center of k-space, resulting in reduction factors of R≈3.89, 8.42 and 16.98), radial sampling and with variable density random sampling patterns that were generated with pdfs from polynomials of different order, as proposed in [1]. For this purpose density powers from 2 to 20 were calculated, and the value with the lowest RMS difference to the original unaccelerated image was selected. Additionally, this experiment was repeated for all acceleration factors from 4 to 20, to illustrate the influence of the design parameter on image quality. Experiments with random patterns were repeated 10 times and mean value and standard deviation of the RMS difference were calculated. Images were reconstructed offline using the IRGN method [2]. Finally, accelerated measurements, again with reduction factors R=4, R=10 and R=18, of the brain of a healthy volunteer were performed with a receive-only 32 channel head coil using the proposed method. A 3D gradient echo sequence (TR/TE=30/6.4ms, FA=13°, matrix: 256x256x256, resolution: 1x1x1mm³) was used which was modified by including a binary 2D mask that defined the randomized 3D subsampled pattern. Raw data was exported from the scanner, a 1D Fourier transform was performed along the readout direction, and partitions orthogonal to this axis were reconstructed offline using the IRGN method.

Results: **Fig. 1** shows the mean RMS errors for reduction factors 4 to 20 for all powers p from 1 to 20 for which the method in [1] was able to generate a pattern without the additional tuning of a second parameter (the radius of a region around the center of k-space where all sample points are included). Since choosing the optimal set of parameters from a two-dimensional search space would have exceeded the scope of this work, and since the ratio of higher to lower frequencies is mainly determined by the polynomial power, we chose to fix this second parameter at its default value. With this choice, patterns for R=4 could only be generated with p>1, while R=20 required p>6. **Fig. 2** shows the results from the downsampling experiments for all investigated sampling patterns for R=10 and R=18. The corresponding RMS differences to a sum of squares (SOS) reconstruction from the original, fully sampled data set are displayed in **Table 1**. It should be noted that the acceleration factors of the regular Cartesian patterns were slightly lower than those of the other sampling trajectories. The reconstruction results from accelerated in vivo measurements using the proposed patterns are shown in **Fig. 3**.

Discussion: Our results show that excellent image quality can be achieved even for very high acceleration factors. The subsampling experiments showed significantly improved image quality in comparison to conventional cartesian and radial scans. Image quality of the proposed method was comparable to the results that were obtained with the pdfs that were generated with the use of polynomials, but it should be noted that extensive optimization of the design parameter had to be carried out in the polynomial case (making use of the fully sampled image, which is not available in practical applications), and that success depends on a suitably chosen mathematical model involving these parameters. As illustrated in Fig. 1, the optimal choice of the design parameter is not straightforward as different values deliver the best results for different reduction factors. The proposed approach ensures that the tuned ratio of low to high sampled frequencies is particularly suited to the type of scan that is performed allowing higher acceleration factors than a fixed ratio. One major advantage is that the method is completely free of any user-defined parameters, and does not require a priori choice of a mathematical model. It is our observation that the method is robust regarding the choice of the reference image, as only a similar distribution of the magnitude of k-space values is required, not similarity in image space (i.e., similar phase as well as magnitude). This distribution does not differ considerably for measurements of the same anatomical region which was already found in [3].
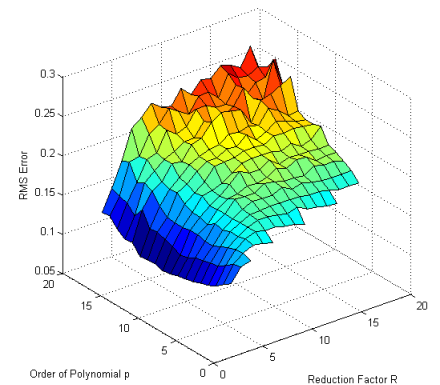

Fig. 1: Influence of the polynomial order p on RMS Errors for reduction factors R=4 to R=20.

Pattern	R=4	R=10	R=18
Cartesian	0.070	0.128	0.255
Radial	0.081	0.137	0.182
Polynomial	0.117±0.002	0.152±0.005	0.154±0.004
Template	0.117±0.003	0.159±0.009	0.168±0.002

Table 1: RMS differences to SOS reconstruction from the fully sampled reference data set for the downsampling experiments of the brain data set. For random sampling patterns, the experiments were repeated 10 times, mean values and standard deviations are shown.
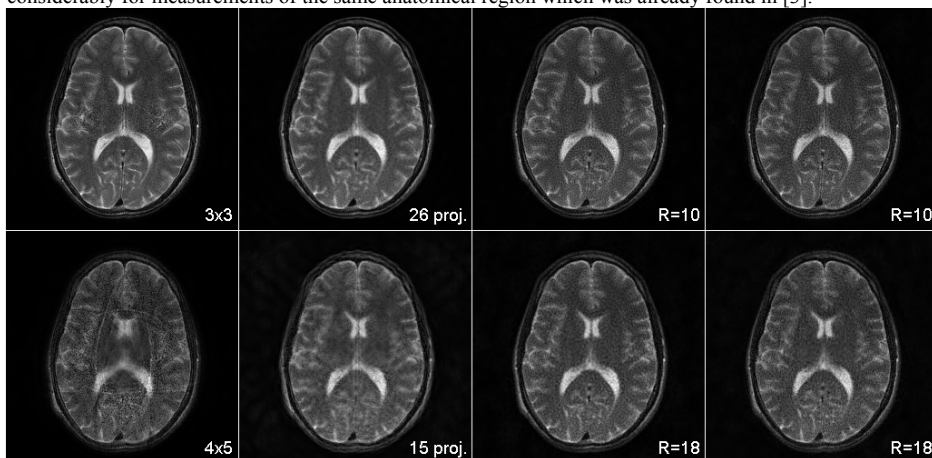

Fig. 2: Results of downsampling experiments of the brain for subsampling with R=10 and R=18. IRGN reconstructions of conventional regular Cartesian subsampling with autocalibration lines at the center of k-space (first column), radial subsampling (second column), variable density random sampling with sampling patterns from polynomial pdfs (third column) and the proposed method (fourth column) are displayed.
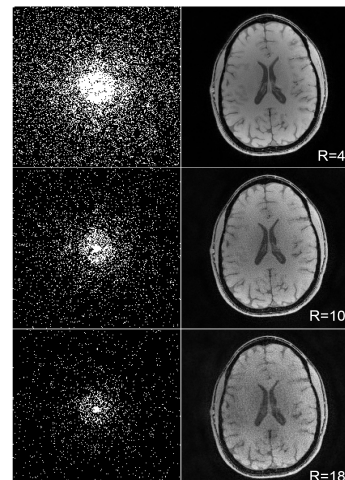

Fig. 3: Results of accelerated in vivo measurements for subsampling with R=4, R=10 and R=18. Used sampling pattern (left), IRGN reconstruction (right).

References:
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[2] Uecker et al., MRM 60: 674-682,
[3] Marseille et al., J. Magn. Reson. B 111(1):70–75 (1996)