Optimally Regularized GRAPPA/GROWL with Experimental Verifications

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Introduction

Generalized auto-calibrating partially parallel acquisition (GRAPPA) [1] has been widely used in many clinical applications. GRAPPA operator for wider radial bands (GROWL) [2] is a fast self-calibrated parallel imaging technique for radial datasets using the GRAPPA formalism. The performance of these techniques can suffer, when the size of auto-calibration signal (ACS) region becomes small. Several previous works have discussed empirical solutions to this problem [3-4]. In this work, an optimal Tikhonov regularization factor is proposed based on an analysis of the condition number for the GRAPPA calibration equation. The technique is applied to both GRAPPA and GROWL, and results show that minimal reconstruction errors are consistently achieved with the proposed method.

Theory

During the GRAPPA calibration process, the data weighting vector w is determined by solving the over-determined linear equation Eqn. (1). Here T_{ACS} and S_{ACS} are vectors of target and source data points from multiple k-space locations and coil channels, collected in the ACS region. The ideal weighting vector, however, should solve Eqn. (2). Here both target and source data points are collected in the *entire* k-space. One key observation is that Eqn. (1) has a much higher condition number than Eqn. (2). The condition number can be computed by applying singular value decomposition (SVD) to the source data matrix S and calculating the ratio between the maximal and minimal singular values (s_{max} and s_{min}). The s_{max} is determined by the overall signal profiles and therefore is independent of the size of ACS region N_{ACS}. For a random matrix, s_{min} can be determined from Eqn. (3) [5]. Here σ is the noise standard deviation for each channel. Therefore a smaller ACS region results in a lower s_{min} value and a higher condition number.

$$T_{ACS} = S_{ACS} w \quad (1)$$

$$T_E = S_E w \quad (2)$$

$$s_{\min} \sim \sqrt{N_{ACS}}\,\sigma \quad (3)$$

$$w_{opt} = \arg\min_w [\|T_{ACS} - S_{ACS} w\|^2 + \lambda^2 \|w\|^2] \quad (4)$$

$$\lambda_{opt} \sim \sqrt{N_E}\,\sigma \quad (5)$$

$$\sigma_{opt} = \sqrt{(N_E - N_{ACS})/N_{ACS}}\,\sigma \quad (6)$$

There are two approaches to artificially increase s_{min}: One approach is to use the Tikhonov regularization [6], which solves Eqn. (1) by minimizing Eqn. (4). Here λ is known as the Tikhonov factor. Our hypothesis is that optimal Tikhonov factor should result in a condition number similar to that of the whole k-space calibration equation (Eqn. (2)). If the entire k-space contains N_E data points, the optimal Tikhonov factor is then Eqn. (5). The other approach is to add random noise to the ACS data to increase σ (Eqn. (3)). Based on the same hypothesis above, the optimal standard deviation of the added noise should be Eqn. (6).

Methods

The proposed regularization scheme was applied to both GRAPPA and GROWL techniques. A noise-free T_1-weighted Cartesian brain MR dataset was downloaded from a simulated brain database (http://www.bic.mni.mcgill.ca/brainweb/). The complex sensitivity profile of a head coil array with eight coil elements equally spaced around a cylinder was computed using an analytic Biot-Savart integration. The k-space data for each individual channel was then derived using the Fourier Transform (FT) for GRAPPA (Cartesian datasets) and inverse gridding for GROWL (radial datasets). Gaussian distributed random noise was then added to both real and imaginary components of each channel of k-space data, resulting in a noise standard deviation in the range of 0.1% - 5.0% of signal intensity of the white matter (the dominant tissue) in the final images reconstructed using the square-root-of-sum-of-square (SSoS) channel combination. Different Tikhonov regularization factors or additive ACS noise levels were applied and results were compared with the noise-free reference using root-mean-square-error (RMSE).

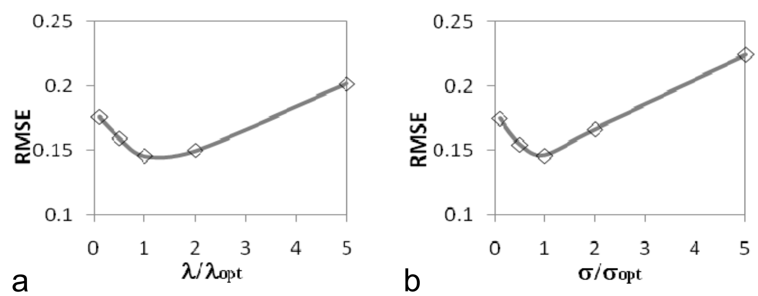

Fig. 1 RMSE for GRAPPA images with different Tikhonov regularization factors (a) and additive noise level in the ACS region (b). λ_{opt} and σ_{opt} are defined in Eqn. (5) and (6), respectively. Image matrix 256 x 256, reduction factor R = 4, ACS line no. = 32, noise level = 5%.

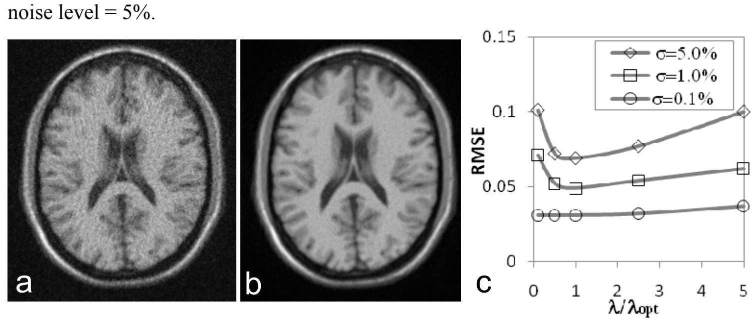

Fig. 2 GROWL results using 32 radial views. (a)-(b): Images (matrix size 256 x 256) without (a) and with (b) the proposed regularization. Noise level = 5%. (c) RMSE vs. regularization factor at various noise levels. λ_{opt} is defined in Eqn. (5).

Results and Discussions

Figure 1 shows the RMSE plot for a GRAPPA dataset (image matrix 256 x 256, reduction factor R = 4, ACS line no. = 32, noise level 5%). The minimal RMSE was achieved either using the proposed Tikhonov regularization factor or the proposed additive ACS noise level. Figure 2a-b shows GROWL images with 32 radial views (noise level 5%). Without regularization (Fig. 2a), residual error is significant due to the small ACS region (20 x 20). With the proposed regularization strategy (Fig. 2b), the RMSE is reduced from 19% to 6.3%. Figure 2c demonstrates that the proposed regularization factors are optimal at all noise levels.

The proposed automatic method achieves an optimal balance between noise and artifact levels by increasing the minimal singular value of the GRAPAP calibration equation. An alternative approach to improve the conditioning of the GRAPPA calibration equation is to reduce the maximal singular value, such as high-passed GRAPPA [7].

References

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