Non-linear Inversion in Parallel MRI: Considerations on Noise Amplification in the Joint Estimation of Image and Coil Sensitivities

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INTRODUCTION - Recently, iterative joint estimation algorithms have been proposed to reconstruct aliasing-free images and coil sensitivities in a single step from self-calibrating sampling trajectories such as Cartesian with variable density (VD) [1, 2]. These algorithms extend parallel imaging reconstruction algorithms based on a sensitivity reference scan [3] or auto-calibration data [4], with the advantage that geometric discrepancies between reference and diagnostic scans are avoided and high-frequency information is exploited to compute the coil sensitivities. However, due to the non-linearity of the reconstruction method, their behavior with respect to noise amplification is more difficult to predict. In this work, we extend the non-linear inversion algorithm (NLINV) of [2] by incorporating the noise covariance of the coil array in the minimization function and by applying additional regularization for the coil sensitivities, both with the aim of improving the SNR of the reconstructed image. We present detailed results on the noise amplification properties of this joint reconstruction scheme and evaluate the proposed algorithm in vivo.

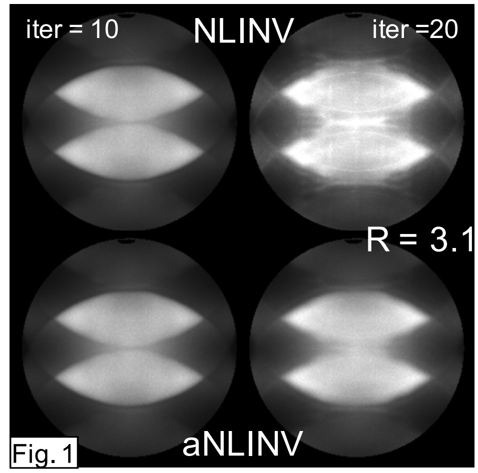

Fig. 1

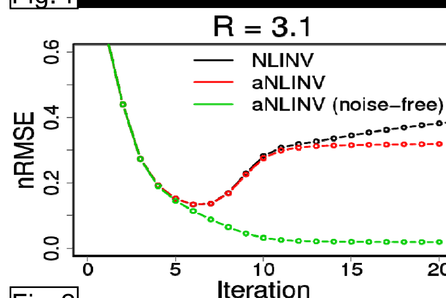

Fig. 2

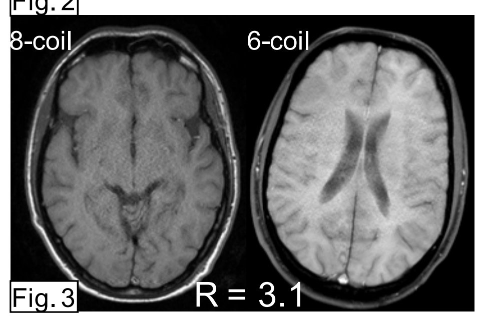

Fig. 3

METHODS - In NLINV, a non-weighted quadratic penalty function is minimized with respect to both image and coil sensitivities by combining the iteratively regularized Gauss Newton method with the conjugate gradient (CG) algorithm. To ensure convergence to a suitable solution, a decreasing level of regularization, based on the Sobolev norm of the coil sensitivities, is applied at each iteration [2]. We propose the following modified update scheme for NLINV, denoted thereafter aNLINV:

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \left(\mathbf{DF}_n^H \Psi^{-1} \mathbf{DF}_n + (\alpha_n + \alpha)\mathbf{I}\right)^{-1} \left(\mathbf{DF}_n^H \Psi^{-1}(\mathbf{y} - F(\mathbf{x}_n)) + (\alpha_n + \alpha)(\mathbf{x}_n - \mathbf{x}_0)\right)$$

In this equation, x_n is the vector of image and coil sensitivity values at all locations after n iterations, F is the encoding operator consisting of multiplication between image and sensitivity, Fourier transform, and projection onto a grid of sampling positions, DF_n is the Jacobian matrix of $F(x_n)$, and Ψ is the noise covariance matrix of the coil array. The use of Ψ as weighting matrix in the quadratic function aims at reconstructing an image with optimal SNR in case of correlated noise in the receive channels [5, 3]. Furthermore, as α_n follows a geometric decay [2], the addition of the constant regularization parameter α ensures a minimum level of regularization even at higher iteration numbers, which prevents strong noise amplification in the coil sensitivity estimates.

The properties of the modified regularization scheme with respect to noise amplification were assessed in Monte-Carlo simulations [6], for two Cartesian trajectories with VD sampling and net acceleration factors of R = 1.8 and R = 3.1. In each case, 400 noisy data sets, synthesized with Gaussian white noise and sensitivity profiles of a 6-element coil, were reconstructed, and noise statistics were calculated for individual pixels. Furthermore, the benefit of additionally exploiting the noise covariance matrix of the coil array was evaluated with human brain data acquired on a 1.5T scanner (Achieva, Philips Healthcare) with 8-element and 6-element head coils. A turbo spin echo sequence was used in combination with VD Cartesian sampling, for different acceleration factors.

RESULTS – Fig. 1 shows the noise standard deviation (SD) maps obtained for R = 3.1 in the simulations. As opposed to NLINV, no significant increase in the SD can be observed with aNLINV between 10 and 20 iterations, suggesting a behavior similar to CG-SENSE with known sensitivities. The evolution of the normalized RMSE with respect to the iteration number in one of the simulated datasets is displayed in Fig. 2. The comparison with the noise-free case clearly demonstrates uptake of noise after approximately 7 iterations. However, the proposed regularization scheme effectively stabilizes noise amplification after 10 iterations, as opposed to NLINV. Two examples of brain images reconstructed with aNLINV are shown in Fig. 3. The noise covariance matrix was computed from noise samples and revealed the existence of significant correlation between channels. By forming the RMSE with a reference image acquired without acceleration, the reduction in total noise amplification achieved by the use of the noise covariance matrix can be quantified: 14.8% (R = 1.8) and 12.8% (R = 3.1) for the 6-element coil acquisition, 8.8% (R = 1.8) and 5.3% (R = 3.1) for the 8-element coil acquisition.

DISCUSSION and CONCLUSIONS - The regularization method applied in aNLINV stabilizes the sensitivity estimates, which limits noise amplification in the image estimates, even at larger iteration numbers. A further improvement in the SNR of the final images can be achieved by decorrelating the channel signals based on the noise covariance matrix of the coil array. Although the number of free parameters in aNLINV is much higher than in regular SENSE, the obtained results suggest that noise amplification is comparable. Further improvement in noise reduction can be achieved if explicit regularization of the image is applied, e.g. based on the L1-norm [7].

REFERENCES - [1] Ying and Sheng, MRM; 57:1196-202 (2007). [2] Uecker et al, MRM; 60:674–82 (2008). [3] Pruessmann et al, MRM; 42:952–62 (1999). [4] Griswold et al, MRM, 47:1202-10 (2002) [4] Pruessmann et al. MRM; 46:638-51 (2001). [5] Roemer et al, MRM, 16:192-225 (1990). [6] Eggers et al., ISMRM, 2429 (2005). [7] Uecker et al., ISMRM, 1479 (2008).