Calibrationless Parallel Imaging Reconstruction by Structured Low-Rank Matrix Completion

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Introduction: Autocalibrating parallel imaging (acPI) methods [1] are known to be robust in practice, producing more diffused artifacts in situations where explicit coil sensitivities are hard to obtain. In these methods, a fully sampled calibration area in k-space must be acquired. However, in some situations, obtaining a sufficiently large calibration can be prohibitive (for example in spectroscopic imaging). Most joint estimation (coil & data) techniques [2-5] still require some calibration. Here we present a new acPI method that does not explicitly require a full calibration area. Instead, the method jointly calibrates, and synthesizes missing data from the entire acquired k-space. The proposed method is based on low-rank matrix completion, which is an extension of the compressed sensing theory to matrices [6].

Theory: ***Low-rank matrix completion*** is a hot research topic and is an extension of compressed sensing to matrices [6]. In general, missing entries of a matrix can be completed if the original matrix has a low-rank and incoherence conditions (randomly undersampled entries) exist. Efficient algorithms for reconstruction are based on singular-value thresholding [6], which we use here.

acPI as low-rank structured matrix completion: GRAPPPA and other acPI methods [1,4,7] exploit linear dependency in k-space. Overlapping blocks in k-space (across coils) are linearly dependent, which enables calibration of GRAPPA interpolation kernels. This means that a Calibration matrix, in which the rows are made of data from overlapping blocks in k-space has low-rank (see fig. 1). Therefore, an (incoherently) undersampled k-space acquisition can be recovered by completing missing entries, which give the lowest rank matrix. The following is a very efficient algorithm based on singular-values thresholding [6] (Fig. 2): (1) Construct matrix A from overlapping blocks (2) Compute $[U,\Sigma,V]$ = svd(A); Threshold the singular values $\Sigma = S(\Sigma,\lambda)$; (3) Compute: $A = U\Sigma V'$ (4) Reconstruct k-space, from A. (5) Impose acquired k-space entries (6) Repeat 1-5 till convergence.

Methods and Results: Data from an SPGR sequence (matrix: 200x200x8 1mm^3 resolution) was undersampled in retrospect by a factor of 3 using a uniform poisson-disc pattern [7]. Overlapping window size was 6x6x8 resulting in matrix A size of 38025x288. A hard singular value threshold function was used choosing the largest 45 (of 288) singular values. The number of iterations was N=30. The results are demonstrated in Fig. 3 showing a good reconstruction with no calibration area at all.

Conclusions: We demonstrated a truly auto-calibrating PI method based on low-rank matrix completion. It can be used to reconstruct undersampled data, or if reduced computation complexity is required, to generate calibration data for GRAPPA-like acPI methods, or just to improve calibration when the calibration area is too small.

References: [1] Griswold et. al MRM 2002;47(6): 1202-10 [2] M. Uecker et. al MRM 2008:60:674-682 [3] Ying et. al MRM 2007:57:1196-1202 [4] Zhao et. al MRM 2008:59:903-7 [5] Dylan et.al Parallel Imaging Workshop 2009 [6] Cai, et.al "A singular value thresholding algorithm for matrix completion." 2008, on line manuscript. [7] Lustig et. al ISMRM'09 pp.378

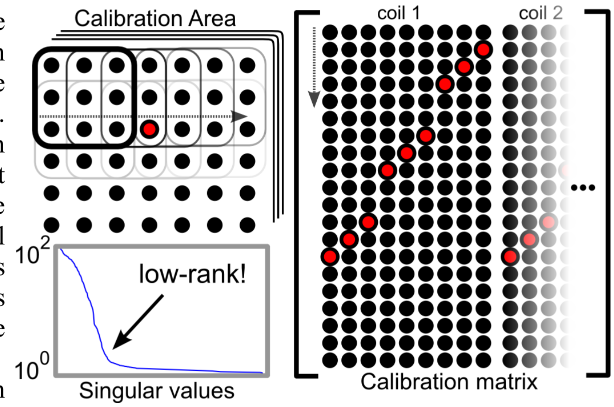

Figure 1: Overlapping blocks in k-space are linearly dependent; therefore a matrix in which rows are made of overlapping blocks has low-rank. It also has a "toeplitz-like" structure (illustrated by red circles). This further reduces the degrees of freedom in a completion problem.

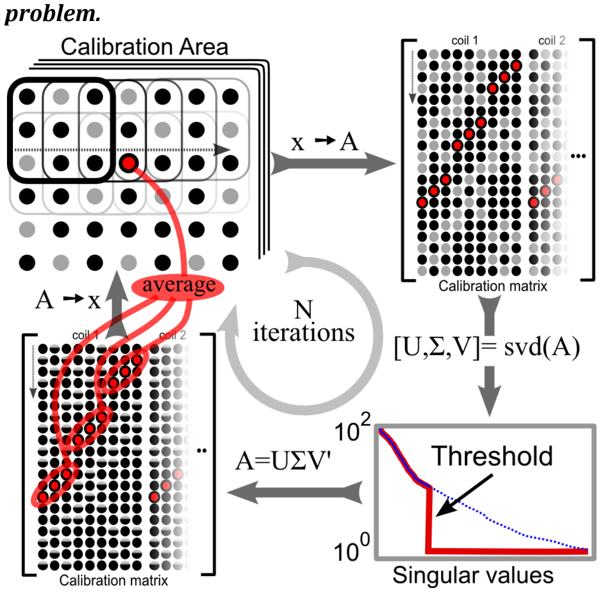

Figure 2: Low-rank matrix completion algorithm. Randomly under-sampled k-space is reordered into a matrix in which rows are made of overlapping blocks. The singular values of the matrix are thresholded and a new matrix is reconstructed. k-Space is then computed while averaging multiple entries. Acquired data is then imposed back for data consistency. This procedure is repeated until convergence.

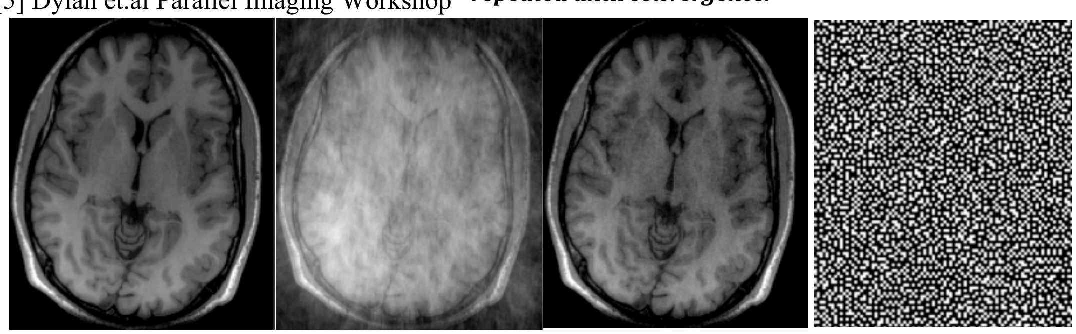

Figure 3: Reconstruction from 3-fold undersampling. Left to right: Fully sampled, zero-filling, low-rank matrix completion and sampling pattern.