Null Space Imaging: a Novel Gradient Encoding Strategy for Highly Efficient Parallel Imaging

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Introduction: The current work proposes a novel encoding scheme that makes use of nonlinear magnetic encoding gradients designed to be maximally complementary to coil sensitivities in parallel MRI. In parallel MRI, multiple receiver coils collect signal locally sensitive to limited regions within the field of view. Coil sensitivity profiles, which map the spatial dependence of electromagnetic signal induction, are used in methods such as SENSE and SMASH to reconstruct images[1,2]. Traditionally, advances in parallel imaging acceleration have not considered the encoding magnetic gradient fields used for localizing spins but have focused more on the receiver coil hardware.

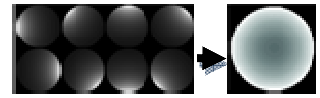

Figure 1. Eight element microstrip array coil

The gradients proposed in the current null space imaging (NSI) approach are designed to be maximally complementary to the receiver coils in order to optimally share the duty of spatial localization in parallel imaging for any imaging application. Recently, PatLoc imaging used multipolar nonlinear gradients to reduce peripheral nerve stimulation during gradient ramping[3]. Whereas PatLoc produces gradients from the conformal mapping $f(z) = z^n$, NSI gradients encode the space that is ill-described by the coil array. Spherical harmonic field gradients approximate the NSI gradients for application during readout in imaging experiments.

Theory: In parallel imaging, the individual coil sensitivity profiles modulate signal from the spin density, leading to differentiation between spatial locations. NSI gradients are designed perpendicular to the coil profiles in order to provide complementary encoding information. The orthogonality constraint specifies that the dot product between coil sensitivity and encoding shape is zero. Furthermore, gradients should encode independent information by being orthogonal to each other. These constraints can be expressed mathematically as:

$$\mathbf{C}_l \cdot \mathbf{G}_m = 0, \forall\, l, m$$

$$\mathbf{G}_i \cdot \mathbf{G}_j = \delta_{ij} \mid i, j \in (1, M)$$

In vector form, C is the coil sensitivity indexed by coil, G_m the magnetic gradient shape applied during an echo, and δ_{ij} is the Kronecker delta. The SVD analyzes the collection of coil profiles to find a complete set of orthogonal vectors.

$$\text{svd}(\mathbf{C}) = \mathbf{U\Sigma V}^H$$

$$\mathbf{V}_0 = \mathbf{V}_{:,L+1:N}$$

C contains the sensitivities stacked in rows and V^+ denotes the transpose conjugate of V. V_0 contains the vectors that have a zero dot product with the coil profiles. In order to generate gradient shapes, the vectors must be interpreted as gradients by taking the phase of the vectors in V_0. Combining the SVD analysis with the orthogonality relation yields the gradient shapes, where the phase of vectors in V_0 generates the gradients G in $e^{-i2\pi\gamma Gt}$. The resulting analysis produces N - K encoding functions. The set of orthogonal gradients must be summarized to a concise set for imaging. Casting the set of orthogonal gradients as a linear operator considers the set of NSI gradients as the encoding matrix for imaging experiment with uniform coil sensitivity. With an SVD, the range space of the gradients can be expressed in an orthogonal basis. A set of proposed gradients is selected from the range space, which are the columns from the SVD.

$$\text{svd}(\mathbf{G}_\perp) = \mathbf{W\Xi X}^H$$

$$\mathbf{G}_m = \mathbf{W}_{:,m} \mid m = 1,2,\ldots,K$$

K gradients are chosen depending on the desired reduction factor, R.

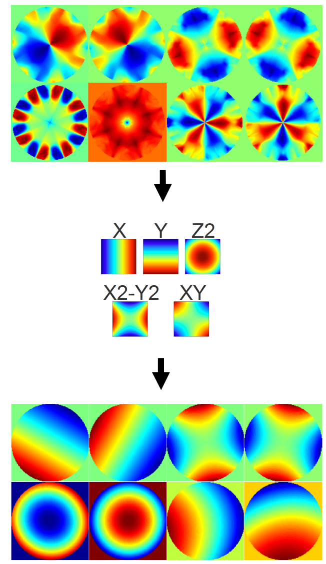

Figure 2. First eight NSI gradients after approximation with second order spherical harmonics

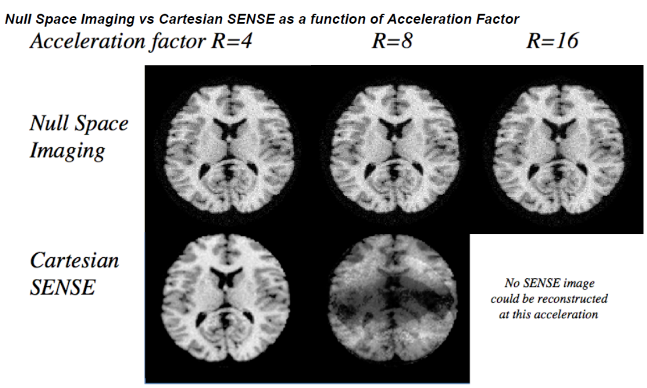

Figure 3. The NSI reconstruction retains high fidelity even as the acceleration factor moves above the number of receiver coil elements.

Method: The sensitivity profiles of eight microarray strip coils circumferentially arranged around a field of view were simulated based on theoretical calculations by Lee[4]. The above theory generated NSI gradients complementary to gradient encoding, which were then approximated with a least squares fit of physically realizable first and second order spherical harmonic gradients. Each gradient shape is used for a single readout of a spin echo sequence.

Simulations under increasing acceleration probed the robustness and behavior of NSI imaging. Invariant to all cases was a spin echo sequence with dephase and rephase lobes for each nonlinear gradient in the set and no phase encoding. NSI reconstructions were compared to SENSE reconstructions at R=4 and R=8 echoes for a 128x128 image. During the acceleration series, the sampling rate was held constant at 1024 samples over an echo length of 5 ms. In addition, a base level of Gaussian noise was added to the phantom. Noise amplification in the SENSE reconstructions was tempered using Tikhonov regularization via a truncated SVD of the aliasing matrix. Reconstructions were performed using the Kaczmarz iterative algorithm, an algebraic reconstruction technique. The results indicate that NSI greatly out-performs conventional Cartesian SENSE achieving acceleration factors higher than the number of receiver coils.

Discussion: By considering the spatial localization abilities of a coil array independent of a reconstruction method, we have designed a novel gradient encoding method that broadly complements the spatial localization provided by the coil array. In addition, the components of the aggregate gradient shape are spherical harmonics, which are well-known, implemented solutions to Laplace's equation. Sequences using NSI gradients enable high scan time acceleration for nearly any imaging experiment and require only a limited set of receivers.

References: [1]Sodickson, DK, Manning, WJ. Med. Phys 1997; 38: 591. [2]Preussman, KP, et. al. Magn. Reson. Med 1999; 42:952. [3]Hennig, et. al. ISMRM 2007, 453 [4]Lee, RF, et. al. Magn. Reson. Med 2004; 51:172.