Phase Contrast Velocity Imaging Using Compressed Sensing

D. J. Holland[1], D. M. Malioutov[2], A. Blake[2], L. F. Gladden[1], and A. J. Sederman[1]
[1]Department of Chemical Engineering and Biotechnology, University of Cambridge, Cambridge, Cambridgeshire, United Kingdom, [2]Microsoft Research Cambridge, Cambridge, United Kingdom

Introduction: Imaging velocity fields using Magnetic Resonance (MR) is time consuming. In this work we show that Compressed Sensing (CS) can be successfully applied to phase encoded velocity MR imaging to decrease data acquisition times by up to an order of magnitude. Compressed Sensing is a recent development from the signal processing field that utilizes the principles behind image compression technology to reconstruct images from sparse **k**-space sampling; to date CS has only been applied to extract intensity images. We have developed a CS algorithm which retains phase information and can therefore be used to measure the phase encoded velocity distribution in both gas- and liquid-flows from significantly under-sampled data in both conventional Cartesian acquisitions and variable density (VD) spirals.

The reconstruction of the sparse **k**-space data was achieved using a CS algorithm based on the approach of Lustig et al. [1]. The image reconstruction method is based on maximizing the sparsity in a transform, Ψ, of the image. Examples of these sparse representations include the wavelet, and the spatial finite differences transforms. The reconstruction is obtained by solving the following constrained optimization problem:

$$\begin{aligned} \text{min} \quad & \|\Psi \mathbf{x}\|_1 \\ \text{subject to} \quad & \|F\mathbf{x} - \mathbf{y}\|_2 < \varepsilon_1 \qquad (1) \\ & \|(1-\mathbf{M}).*\mathbf{x}\|_2 < \varepsilon_2 \end{aligned}$$

where **x** is a vector representation of the image, F is the Fourier transform operator, **y** is the vector of the measured **k**-space data points and **M** is a mask describing where the signal is expected to be. The constraints, ε_1 and ε_2, are thresholds that can be set to the expected noise level. The l_1-norm acts as a proxy for sparsity – i.e. minimizing the objective in Equation 1 produces an image which has the sparsest representation in the transform domain while remaining consistent with the acquired measurements.

Methods: Experiments were performed using a model system consisting of a 27 mm diameter cylinder filled with 5 mm diameter glass spheres. Water or sulphur hexafluoride gas was flowed through the cylinder at flow rates between 0 and 17 ml s^{-1}. Data were acquired using a spin echo velocity imaging sequence and were undersampled by factors of 2 to 5. Variable density spiral data were also acquired. All data were acquired on a Bruker DMX200 vertical magnet.

Results: Figure 1 shows a velocity field reconstructed from a 30 % sampling of a Cartesian **k**-space using Equation 1. The CS reconstruction has been used, on both simulated and real data, to reconstruct images from only 30 % of the full **k**-space data set with less than 1 % error in the total flow measurement and less than 9 % error based on the ℓ_2-norm. Furthermore, we demonstrate that the resulting velocity distribution is unbiased. By combining this approach with variable density spiral acquisitions it has been possible to achieve a full order of magnitude decrease in imaging time compared with conventional Cartesian imaging.

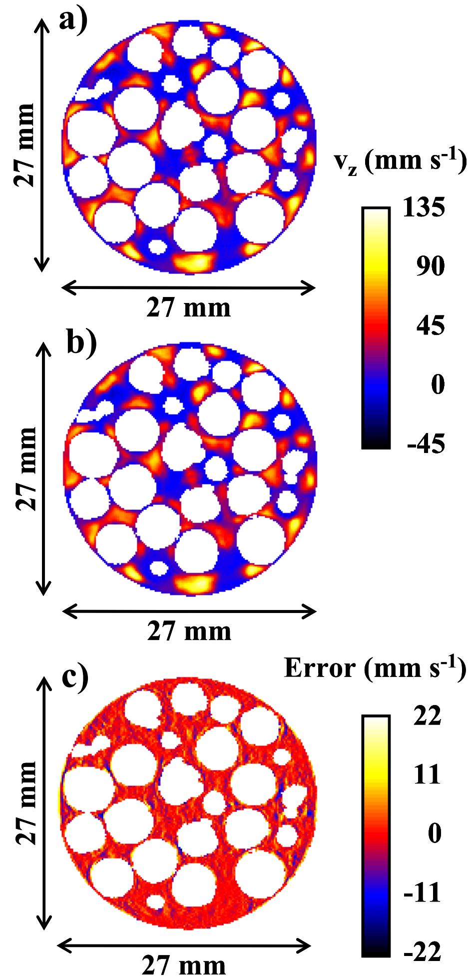

Figure 1: a) MR map of the axial velocity of water flowing through a packed bed of 5 mm diameter spheres for full **k**-space sampling. b) Reconstruction of the velocity field in (a) with only 30 % sampling of the full **k**-space distribution. c) Absolute error in the reconstructed velocity distribution.

Discussion/Conclusions: The CS velocity imaging approach described in this work has been demonstrated to reconstruct images from only 30 % of the full **k**-space data with negligible error. Furthermore, acceleration factors of up to 5 have been achieved with an error of < 20 %, which corresponds to the limit of a visually detectable distortion to the image. The approach outlined here has been demonstrated on a model system, but it is equally applicable to applications in MR angiography.

References: [1] M. Lustig, D. Donoho, J.M. Pauly. Magn. Reson. Med. 58 1182–1195 (2007).