On the Form of the Residue Function for Brain Tissue

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Introduction

Dynamic susceptibility weighted (DSC) perfusion MRI is a widely applied technique in the diagnosis of ischemic stroke. Basing on the tracer kinetic model [1], the hemodynamic parameters blood volume (CBV), blood flow (CBF) and mean transit time (MTT) can be computed by deconvolution of the tracer kinetic equation ct = CBF (ca*R) where * denotes the convolution, ct is the tissue concentration and ca the arterial input. CBF and MTT can be determined from the residue function R(t) that characterizes the tissue. Being an ill-posed problem, the deconvolution requires a model free regularization approach or alternatively the assumption of a functional form of the residue function which allows an optimization of the functions parameters. Mouridsen et al have presented [2] such a model-based approach using incomplete gamma functions that enables an interpolation between the commonly used exponential and box-car function that characterize two extreme cases of tissue compositions. Studies carried out by using ASL [3] however indicate that the use gamma variate functions for the parameterization of the residue function is not correct. In this work we present the framework from which the form of the residue function can be derived from the laws of laminar flow and a vascular tree model.

Theory

Laminar Flow. The blood flow in the brain vasculature can be assumed to be laminar. The vessel segments between bifurcations are assumed to be cylindrical pipes, a situation in which the radial velocity distribution is well known. From this, the transport function, which is the distribution of the transport time trough the pipe takes the form $h(t) = t0/t^2$ for $t>t_0$, where t_0 is the transit time of blood in the central streamline. The transport trough a network of connected pipes is then given by a chain of convolutions of h(t) for each segment yielding the total transport function.

Scaling Rule. According to the ideas described in [4], the arterial tree is self-similar and obeys Murray's Law. Assuming the splitting in two equal vessels, their size is reduced by the factor $2^{1/3}$ at each bifurcation. So do the segment lengths, l, and the central velocity, v_0 keeping t_0 invariant.

Residue Function. The vascular system is modeled to consist of vessels between r_{max} = 1.5 mm and r_{min} = 3.7 µm (26 levels of the vascular tree = N_{max}). The proportionality constant between the vessel radius and its length was estimated to be 40. The minimal transit time, t_0=50ms.

The apparent residue function R_{tot} as measured by deconvolution between the tissue curve and the global arterial input function (AIF) is composed of two contributions, the transport through the vasculature (h_{tree}) to the given site and the transport through the voxel at this site (R_{true}). The voxel is defined to entirely contain the complete segment of the vascular tree, whose total length is smaller than half the voxel size. The venous tree is assumed to be symmetric to the arterial tree. This defines the number N_{vox} from which on the following sub-tree is contained in the voxel. Thus $R_{tot} = h_{tree} \otimes \left(1 - \int_0^t {}^{\otimes}\prod_i h_i(t')dt'\right) := h_{tree} \otimes \left(1 - \int_0^t h_{eff}(t')dt'\right)$ where $i \in \{N_{vox} \ldots N_{max}\}$ and the product ${}^{\otimes}\prod_i$ denotes a convolution chain. h_{eff} is the effective transport function through the voxel.

Results and Dicusssion

For a voxel of size (2x2x2)mm N_{vox} = 19. The evolution of h_{tree} is shown in Fig. 1. As modeled above, R_{true} is shown in Fig 2. h_{tree} acts as a filter on R_{true} and yields R_{tot} as depicted in Fig. 2 which results in the known underestimation of blood flow by choosing a global AIF. With MTT being the expectation value of h_{eff}, its definition turns out to be dependent on the acquisition time, as the integral $\int h_{eff}(t) \cdot t \, dt$ does not converge. In practical implementation, the details of data processing define the regularization scheme of the integral. In order to enable an MTT determination in this study, the integral is defined as the area under the residue function for a given long time (1000 s). Being also the area under the residue function, MTT is not influenced by h_{tree}, that is normalized to unit area by definition. If an ischemic stroke is modeled to reduce the velocities from the generation of the occlusion for the whole sub-tree by 80% the MTT is strongly increased where CBF is estimated more correctly (Fig. 3). This is due to the fact that the time spent within the voxel is strongly increased.

From this framework one can understand that for small voxels, N_{vox} is larger, which leads to an increased contribution of h_{tree} that leaves MTT unchanged but leads to underestimation of CBF. In this case the sub-tree inside the voxel has a shorter plateau and leads to a function that can be confounded with an exponential function as often found in simulations for a well mixed compartment. However the nature of the laminar transport results in a slowly decaying tail of the function.

The model for the transport functions presented here is in an excellent agreement with ASL measurements (this result is presented in another abstract).

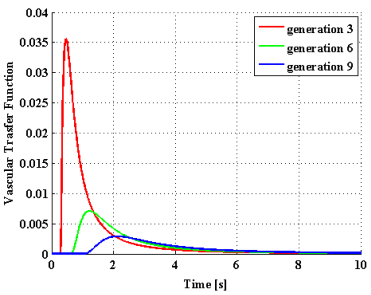

Fig. 1: Evolution of the transport function after 3, 6 and 9 generations.

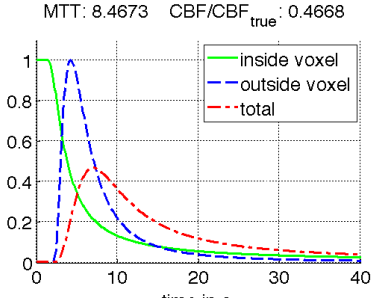

Fig. 2: R_{true} (green), h_{tree} (blue) scaled to 1 and R_{tot} for normal tissue.

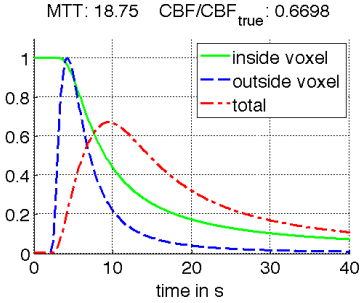

Fig. 3: R_{true} (green), h_{tree} (blue) scaled to 1 and R_{tot} in the case of ischemic stroke.

References: [1] Ostergaard: MRM 36 (96) [2] Mouridsen: NeuroImage 33 (2006) [3] Gall: ISMRM 2008 [4] R. Turner: NeuroImage 16 (2002)