

# Design of a data driven deconvolution filter for DSC perfusion

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## Introduction

Bolus tracking perfusion evaluation relies on the deconvolution of the tracer concentration time-courses in an arterial and a tissue voxel following the tracer kinetic model:  $c(t) = f R(t) \otimes a(t)$ , where  $c(t)$  is the concentration in tissue,  $f$  the cerebral blood flow (CBF),  $R(t)$  the residue function and  $a(t)$  the arterial input function (AIF). The deconvolution is most commonly performed using Fourier or circular singular value decomposition (oSVD). Being an ill posed problem deconvolution relies on regularization. The two methods are mathematically equivalent, and a transformation of the regularization filters between them is defined. It has been shown that a regularization threshold for oSVD has to be adaptive to the properties of the input data such as noise [1, 2]. In this work we employ Tikhonov regularization in the Fourier domain to design a data driven, smooth filter that only depends on one parameter  $\lambda$ . The dependence of the optimal regularization parameter  $\lambda_{opt}$  on the contrast to noise ratio (CNR) of the arterial input function (CNRa), the CNR of the tissue concentration (CNRt) and the first moment difference ( $\Delta m_1$ ) of the two time courses is found using simulations. The resulting filter is applied to simulated and measured bolus tracking perfusion data and compared to oSVD deconvolution with a constant threshold. Exploiting the equivalence of the Fourier and oSVD approach, the filter is transformed and compared to the corresponding oSVD thresholds.

## Methods

**CNR Definition.** In this work CNR was defined to be the ratio of the average amplitude of the relaxation time-course during the first bolus passage [3] and the noise amplitude during the relaxation time-course baseline. The integral character of those measures ensures the numerical stability of the CNR computation.

**Filter Design.** A Tikhonov filter that penalizes oscillations in Fourier domain takes the form  $H = 1/(1 + \lambda^2 * w^4 / (abs(a(w))^2))$ .  $\lambda_{opt}$  was defined using an optimum criterion minimizing the sum of squares of the systematic error introduced by the filter itself and statistical error still present after filtering.

**Simulations.** In order to find the dependence of  $\lambda_{opt}$  on CNRa, CNRt and  $\Delta m_1$  1024 noise realizations for each given point in the parameter space were calculated. White Gaussian noise was added to the signals and CNR computed from the noisy relaxation time-courses. The signals were modeled to be  $s(t) = s_0 * \exp(-\Delta R(t) * TE)$ . For the arterial signal  $\Delta R(t)$  was modeled using the gamma variate function [4] with  $\alpha = 3.0$  and  $\beta = 1.5$ . The time interval covered by the simulations was 100s and TR 1.5s. The tissue signal was found by convolution of the corresponding relaxation time-course with an exponential residue function at varying CBF between 10 and 90 ml/100g/min and a constant blood volume fraction of 0.04. Using this data a hyperplane  $\tau_1 \cdot CNRa + \tau_2 \cdot CNRt + \tau_3 \cdot \Delta m_1 + \ln(\lambda) + \tau_4 = 0$  was fitted.

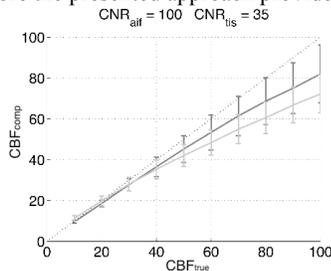
**MRI Measurement.** A dataset from a young and healthy volunteer was reexamined (approved by the local ethics committee). The measurement was performed on a 3 T clinical scanner (TRIO, Siemens, Erlangen, Germany). Images were acquired with a single-shot gradient echo - spin echo sequence with two echo planar readouts with the echo times  $TE_{GE} = 25ms$  and  $TE_{SE} = 85ms$ ,  $T_R = 1800ms$ , matrix size 88 x 88, 16 slices (slice thickness 4mm at 25% interslice gap, pixel size 4 x 4mm<sup>2</sup>), 50 frames. Gd-DTPA (Multihance) at a dose of 0.2ml per kg body weight was automatically injected (5ml/s) followed by a 20ml saline flush. The contrast injection was started with 10s delay.

## Results

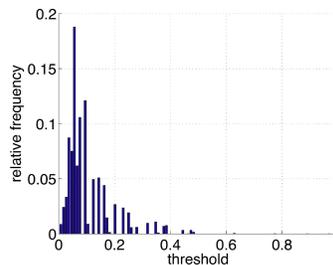
From the simulations the parameters of the hyperplane are found to be:  $\tau_1 = -0.04$ ,  $\tau_2 = 0.03$ ,  $\tau_3 = -0.55$  and  $\tau_4 = 8.50$ . The proposed method shows a smaller flow underestimation especially at high flows compared to the values of oSVD using a fixed threshold of 0.03 (Fig. 1). At the same time the uncertainty of these values is increased due to the stronger consideration of the introduced statistical error (Fig. 1). Exploiting the equivalence of Fourier and oSVD the Tikhonov parameters of the measured data set translate to thresholds in oSVD yielding an expectation value of 0.08 +/- 0.05 (Fig. 2), which agrees well with the values between 0.03 and 0.1 in [1]. An example of the normalized FFT-spectrum from one voxel and the filter is shown in Fig. 3.

## Discussion

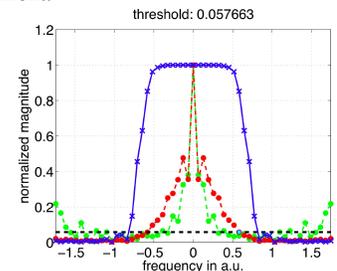
The proposed method is a comprehensive approach for the design of data-driven filters that can be easily adapted to specific needs. The choice of the optimization criterion used in this work show an improvement in the systematic error of the estimated flow as compared to oSVD at a constant threshold. The parameters for the  $\lambda_{opt}$  can be robustly computed from the input data. Application to measured data is in good agreement with [1], furthermore the presented approach provides a deeper insight in the filters action and is simple to implement.



**Fig 1:** CBF computed by the oSVD approach (light gray) and by the proposed approach (dark gray) versus the true CBF used in the simulation. The error bars mark one standard deviation around the mean. The black dotted line marks the identity.



**Fig. 2:** Optimal oSVD threshold distribution derived using  $\lambda_{opt}$ .



**Fig. 3:** Example of normalized FFT-spectra of the AIF (red) and the tissue (green). The optimized filter is plotted in blue. The corresponding oSVD threshold is represented by the black dashed line.

**References:** [1] Wu, O.: MRM 50 (2003) [2] Liu, H.: MRM 42 (1999) [3] Gall, P.: Magma (2009) [4] Ostergaard, L.: MRM 36 (1996)