Towards an optimal distribution of b-values for IVIM imaging

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Introduction:
The intra voxel incoherent motion (IVIM) theory is a sophisticated method to separate diffusion and perfusion effects using diffusion weighted imaging [1], which is particularly useful for the investigation of abdominal lesions [2,3]. However the quality of the parametric maps, especially of the perfusion related maps, is usually limited due to large uncertainties of the fitted parameters [4], in particular when clinical feasible scan times are used. The choice of the number and distribution of b-values varies greatly [2,4] and a fundamental basis for this choice is currently not available. Therefore, the aim of this work was to find an optimal distribution of b-values. To this end we used Monte Carlo simulations where the reduction of the error of the fit was used as criterion to find this distribution.

Materials and Methods:
The signal curve was generated according to the IVIM theory (equation 1) using three different parameter sets of perfusion fraction f, diffusion coefficient D and pseudo diffusion coefficient D*. The values used for the three parameter sets were taken from the literature for brain, liver and kidney (brain: f=5 %, D=1 µm^2/ms, D*=10 µm^2/ms [5]; liver: f=30 %, D=1 µm^2/ms, D*=60 µm^2/ms [2]; kidney: f=30 %, D=1.5 µm^2/ms, D*=15 µm^2/ms [6]; For every b-value, Gaussian noise was added to the complex signal to simulate a Rician distribution of the magnitude signal. Equation 1 was then fitted to the noisy signal curve using the Levenberg-Marquardt algorithm. Initial values were f=10 %, D=0.0001 s/mm² and D*=0.001 s/mm². No limiting value was implemented. The complete process was repeated 5000 times for each set of IVIM parameters and b-values and the relative overall error σ (equation 2) were calculated.
Starting from an initial b-value distribution {b=0, 40, 1200 s/mm²}, the σ of all distributions with the initial three b-values plus one additional b-value in the range from 0 to 1400 s/mm² (in steps of 10 s/mm²) were tested. The distribution with the minimal σ was then chosen as optimal new b-value distribution. This process was consecutively iterated to obtain distributions with up to 100 b-values. The optimization was run four times: three times for the individual organs and once with the summed σ of all three organs yielding the distribution {b_{sum}}. All simulations were implemented in C++.

$$S_b = S_0\left((1-f)\cdot\exp(-bD)+f\cdot\exp(-b\cdot(D+D*))\right)$$

Eq. 1: IVIM-equation: f, D and D* are the fitting parameters and S_b is the MRI signal as a function of the b-value.

$$\sigma = \frac{1}{5000}\left(\frac{\sqrt{\sum_{i=1}^{5000}(f_i - f)^2}}{f} + \frac{\sqrt{\sum_{i=1}^{5000}(D_i - D)^2}}{D} + \frac{\sqrt{\sum_{i=1}^{5000}(D^*_i - D^*)^2}}{D^*}\right)$$

Eq. 2: Relative overall error, where f_i, D_i, D_i^* are the fitted results of the i-th repetition.

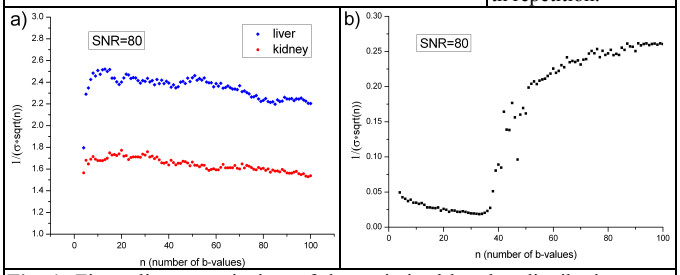

Fig. 1: Fit quality per unit time of the optimized b-value distributions as a function of the number of b-values at SNR=80 for the liver and kidney parameters (a) and the brain parameters (b)

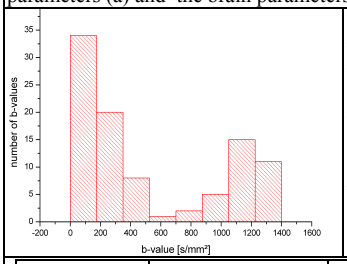

Fig. 2: Histogram of {b_{all}} showing the distribution of b-values for n=100. For the optimal measurement of the IVIM-parameters many b-values in the range of 0 to 360 and around 1200 should be measured. Sampling of the mid range is less beneficial.

	b_{sum}	b_{lit}	b_{av}
brain	σ=16.3 %	σ=92.7 %	σ=63.6 %
liver	σ=0.81 %	σ=1.07 %	σ=0.82 %
kidney	σ=1.20 %	σ=1.58 %	σ=1.32 %

Tab.1: Comparison of the relative overall error of the optimized b-value distribution b_{all} to a previously reported distribution b_{lit} and an averaged distribution b_{av} at SNR=200. The error decreases significantly using the optimized b-value distribution b_{sum} compared to b_{lit} for all parameter sets.

Results:
Fit quality per unit time defined as $1/(\sigma\sqrt{n})$ is shown in figure 1 of the optimized b-value distributions as a function of the number of b-values *n* for the three parameter sets at an SNR of 80. Figure 1a shows that the fit quality for the liver increases until 10 b-values are used {b=0,40,1200,10,100,170,20,1260,20,0, in order of importance}. After this point, additional time can be invested either in repetition of this distribution or in additional b-values without strong differences in the fit quality per unit time. For the kidney, this point is reached even faster, at about eight b-values {b=0,40,1200,240,1030,40,240,1070}. The fit quality for the brain parameters shows a special feature (figure 1b): using few b-values, the fit quality per unit time decreases until a critical point is reached at about *n*=40. In this *n* range, the optimization algorithm finds that averaging the three initial b-values is beneficial compared to the addition of novel b-values. Beyond *n*>40 addition of novel b-values improves the fit quality considerably. Figure 2 shows the distribution of b_{sum} (*n*=100). In order to compare this optimized b-value distribution with a previously reported distribution of 16 b-values {b_{lit}=0,10,20,30,40,50,60,70,80,90,100,150,200,400,800,1000} [2] and a distribution that is averaged four times {b_{av}=4*(0,40,100,1200)}, we applied the first 16 b-values of b_{sum} {0,40,1200,200,20,360,1380,40,280,1300,350,0,70,1020,310,80} and analyzed the relative overall error σ for each distribution (see table 1). The error of b_{sum} is 82 % lower for the brain parameters, 24 % lower for the liver parameters and 24 % lower for the kidney parameters compared to b_{lit} and 74 %, 1% and 9 % lower compared to b_{av}.

Discussion:
This work shows that an optimal choice of b-values can substantially minimize the overall measurement error and thus is of critical importance. The presented distribution b_{sum} can be seen as a first step towards an optimal b-value set up for IVIM imaging. Further evaluation is warranted to solidify these results. Furthermore, our results confirm findings of Pekar [5], which indicated that an extremely high SNR is needed for IVIM of the brain. On the other hand, IVIM of the abdomen can be performed at moderate SNR with a low overall measurement error.

References:
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