Compressed Sensing based Diffusion Spectrum Imaging

N. Lee[1], and M. Singh[2,3]
[1]Biomedical Engieering, University of Southern California, Los angeles, CA, United States, [2]Biomedical Engineering, [3]Radiology, University of Southern California

Introduction: Several methods are currently used to resolve multiple fibers within a voxel including Q-space imaging such as DSI and QBI. These approaches are generally limited by the burden of dense sampling in Q-space to avoid aliasing, which makes their application to clinical studies difficult. However, compressed sensing (CS) can attain almost perfect reconstruction from incomplete random Fourier samples of the PDF if they satisfy sparsity and incoherence conditions. The sparsity condition is met if the PDF can be expanded in an orthogonal basis set using few coefficients. The incoherency condition indicates that the reconstruction basis and the sensing basis (which is the Fourier kernel in this case) should be as different as possible to reduce the number of Fourier samples [1]. The objective of this work was to investigate a novel Q-space imaging method, which we call compressed sensing diffusion spectrum imaging (CS-DSI) that applies the CS approach to the reconstruction of the probability density function (PDF) from which the orientation distribution function (ODF) is calculated.

Method: This method extends the idea of CS on 2D image-k-space domain [2] to 3D Q-space-PDF domain where the measured diffusion MR signal E(q) is related to the PDF P(r) by the Fourier transform, $F[E(q)] = P(r)$ [3]. To satisfy the sparsity and incoherency requirements, 3D Wavelet transform is chosen as the sparse basis and a variable-density random sampling pattern within a NxNxN Cartesian lattice that results in the minimum TPSF (Transfer Point Spread Function) is selected to maximize the incoherence [2]. To evaluate the accuracy of the reconstruction method, a simulation study was performed by generating synthetic diffusion signals using a two-Gaussian mixture model and the reconstructed ODFs at various inter fiber angles were compared to the simulation model.

Theory: Let diffusion displacement PDF is indicated as $p \in \Re^n, n = N^3$,where N is the dimension of Cartesian lattice. Let $\Psi = [\psi_1 \psi_2 \cdots \psi_n]$ and $\Phi = [\varphi_1 \varphi_2 \cdots \varphi_n]$ be the reconstruction and sensing basis which is the wavelet and Fourier basis. We express p(r) as a linear combination of an orthonormal basis set Ψ as or $p = \Psi x$. We assume that PDF is K-sparse in wavelet domain indicating that only K coefficients are nonzero. In matrix notation, $F[E(q)] = P(r)$ is written as $p = \Phi^H e$. Thus $e = \Phi p$ and since we observe only a subset of coefficients the equation becomes $e = R_\Omega \Phi p$ with R_Ω extracting the sampled coefficients where $\Omega = \{1 \ldots m\}, m < n$. The final equation is given as $e = R_\Omega \Phi \Psi x$. This underdetermined system has a unique solution if x is sparse and the solution can be obtained by solving the following problem $\min_x \left\| e - R_\Omega \Phi \Psi x \right\|_1$.

A noiseless two-Gaussian mixture model in slow exchange with Δ / δ = 66ms/66ms and equal mixing ratio was used to generate HARDI signal for a voxel using parameters of the real DSI experiment [4]. The eigenvalues for each fiber population were set to {1.7, 0.3, 0.3} um^2/sec based on the nominal value observed in diffusion measurements of white matter axon fibers [5]. On a 11x11x11 Cartesian grid, total 1331=N^3 q-space sampling points were generated with lattice spacing dq=70rad/sec=111.4cm-1, spatial resolution 2pi/[(N-1)dq] =9um, FOV 2pi/dq = 90um, and bmax=5390sec/mm^2. Acceleration factor of 10 was used and the ODFs of full samples and CS-DSI were calculated at different inter fiber angles. For each angle different sampling patterns were used to generate the PDF. Daubechies wavelet with vanishing moment of 4 was used for all reconstructions.

Results:

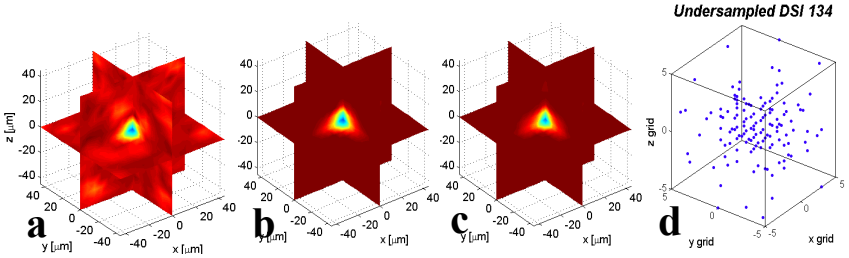
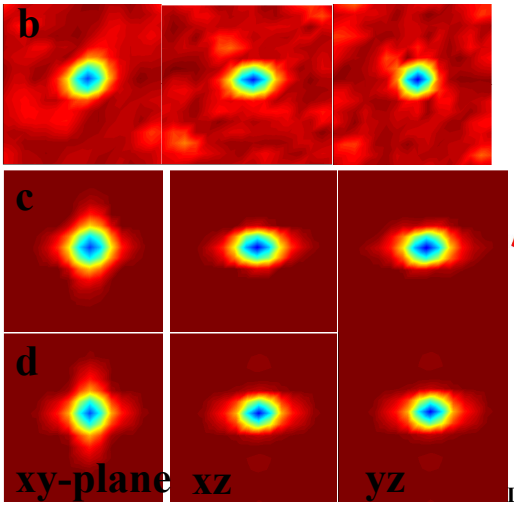


Figure 1. (a) Reconstructed PDF by (b) undersampled data (c) CS (d) full Q-space n =1331 samples.(d) Random sampling pattern with acceleration factor of 10.



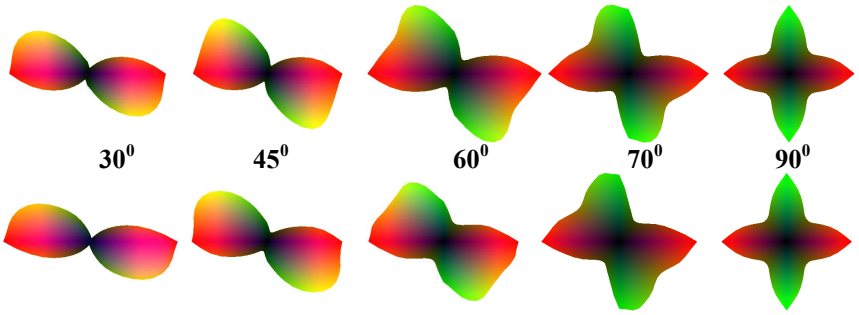


Figure 2.The ODFs reconstructed from the full samples (top) and CS method (bottom). For each angle different sampling patterns were used to generate the PDF.

Discussion and Conclusion: Our results suggest that CS based reconstruction with acceleration factor of 10 gives a well approximated ODF. But this was achievable in the noiseless case where the real human MR signal suffers from a low SNR. This method can be extended using different sparse domains. With the validation of this method using human data, the CS-DSI method enables the application of Q-spacing methods in clinical studies, which shows a promising solution to multiple fiber crossing problem.**Reference**:[1] Emmanuel Candes et al. Inverse Prob 2007,23 969-985.[2]Lustig et al.,MRM 2007,58(6):1182-95 [3] Callaghan, Oxford 1991. [4] Wedeen VJ et al. ISMRM 2000, p82. [5] Tuch DS et al. MRM 2002,48:577-582.