About the origins of diffusion-weighting due to the non-linear phase dispersion induced by frequency-swept pulses

J. Valette[1,2], D. Le Bihan[2], and F. Lethimonnier[2]
[1]CEA-MIRCen, Fontenay-aux-Roses, France, [2]CEA-NeuroSpin, Gif-sur-Yvette, France

Introduction

Adiabatic pulses provide an efficient way to perform broadband and homogeneous magnetization flip, even with strong B_1 inhomogeneities [1]. This property has for example been exploited to perform spectroscopic volume selection, such as in the LASER sequence [2], or slice selective imaging sequence [3]. However, when applied in conjunction with a slice selective gradient, an adiabatic pulse generates a non-linear phase throughout the selected slice along the direction of the gradient. This is due to the frequency-modulated nature of adiabatic pulses, where the magnetization is flipped when the frequency of the pulse is equal to the Larmor frequency Ω. Two components can be identified in this non-linear phase: the phase acquired in the slice selection gradient, depending on the gradient strength and on the instant of the flip t_Ω, and the phase induced by the $\mathbf{B_1}$ field orientation itself, depending on the $\mathbf{B_1}$ phase at the instant of the flip (Fig. 1). This phase dispersion can then be refocused by a second, identical slice selective pulse. It has been recognized that the phase dispersion induced by an adiabatic pulse might induce diffusion weighting. In their pioneer work [4], Sun and Bartha proposed an expression for diffusion-weighting induced by trains of hyperbolic secant pulses, assuming a quadratic phase dispersion (such as induced by a CHIRP pulse), and omitting the contribution of the $\mathbf{B_1}$ phase variations. In the present work, we propose to revisit the origins of the non-linear phase dispersion induced by frequency-swept pulses in order to assess whether the phase variation of the $\mathbf{B_1}$ field during the sweep should be explicitly considered when calculating diffusion-weighting. An analytical expression is then derived for diffusion-weighting induced by a pair of slice selective hyperbolic secant pulses. This expression is validated by numerical simulation of the Bloch equations including diffusion.

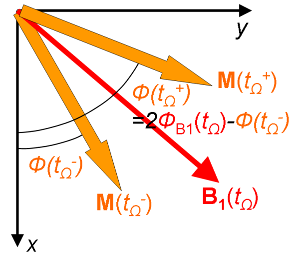

Fig.1: *Evolution of the transverse magnetization M at the instant of the flip t_Ω during a frequency-swept pulse.*

Theory

Non-linear phase seen as a locally linear phase: In the following we will use the usual definition of $k(t)$ as a B_0 gradient momentum. Using this notation, the phase induced by a gradient is $\Phi(x,t)=k(t)\cdot x$. Let's now consider a frequency-swept pulse inducing a non-linear phase $\Phi(x)$ at the instant t_Ω where spins of Larmor frequency $\Omega=\gamma\cdot G_{slice}\cdot x$ are flipped. The phase gradient $\partial\Phi/\partial x$ can be considered linear over the distance experienced by diffusing spins during the sequence, and the effect of diffusion will be to scramble phase and induce signal loss, exactly as induced by a B_0 gradient. This formal analogy between a B_0 gradient momentum k and a phase gradient $\partial\Phi/\partial x$ can be taken advantage of for the evaluation of the effect of the $\mathbf{B_1}$ field phase for an arbitrary frequency-swept pulse, as performed below.

The phase acquired during a frequency-swept pulse: During the pulse of duration T_p, assuming the slice selection gradient is turned on only during the pulse, the phase evolution for magnetization having Larmor frequency $\Omega=\gamma\cdot G_{slice}\cdot x$ and flipped at t_Ω is given by [5,6], Φ_{B1} being the phase of the $\mathbf{B_1}$ field:

$$\begin{aligned} &0<t<t_\Omega : \phi(x,t)=\gamma G_{slice}\cdot t\cdot x \\ &t_\Omega<t<T_p : \phi(x,t)=2\phi_{B1}(t_\Omega)+\gamma G_{slice}\cdot(t-2t_\Omega)\cdot x \end{aligned} \quad [1]$$

The phase gradient evolution during the pulse is then given by:

$$\begin{aligned} &0<t<t_\Omega : \partial\phi/\partial x=\gamma G_{slice}\cdot t=k_{slice}(t) \\ &t_\Omega<t<T_p : \partial\phi/\partial x=\underbrace{2\,\partial\phi_{B1}/\partial t(t_\Omega)\cdot\partial t_\Omega/\partial x-2\gamma G_{slice}\cdot x\cdot\partial t_\Omega/\partial x}_{k_{B1}}+\underbrace{\gamma G_{slice}\cdot(t-2t_\Omega)}_{k_{slice}(t)} \end{aligned} \quad [2]$$

In Eq.[2] two components can be identified: the usual slice gradient momentum k_{slice} (whose sign is changed at t_Ω), and a "radiofrequency" term k_{B1}. However, during frequency-swept pulses, the magnetization is flipped when the pulse frequency is equal to Ω, which can be written as $\partial\Phi_{B1}/\partial t(t_\Omega)=\gamma\cdot G_{slice}\cdot x$. Inserting in Eq.[2] yields:

$$\begin{aligned} &0<t<t_\Omega : \partial\phi/\partial x=\gamma G_{slice}\cdot t=k_{slice}(t) \\ &t_\Omega<t<T_p : \partial\phi/\partial x=\gamma G_{slice}\cdot(t-2t_\Omega)=k_{slice}(t) \end{aligned} \quad [3]$$

In the end, the contribution of the $\mathbf{B_1}$ field orientation is cancelled out when calculating the spatial derivative of the non-linear phase, so that only the phase induced by the slice selection gradient needs to be considered for diffusion-weighting.

Diffusion-weighing during a pair of slice selective frequency-swept pulses: Following the previous analysis, diffusion weighting induced during a pair of slice selective frequency-swept pulses (excluding potential spoiler gradients) is simply the integral of $k_{slice}(t)^2$, which can be expressed as a function of t_Ω. Introducing $\alpha=2t_\Omega/T_p-1$ ($-1<\alpha<1$), and Δ the delay between the two pulses, integration of $k_{slice}(t)^2$ yields:

$$b=\gamma^2 G_{slice}{}^2 T_p{}^2\left(\left(\left(1-\alpha^2\right)/2-1/3\right)T_p+\alpha^2\Delta\right) \quad [4]$$

The above equation allows the evaluation of b as a function of the position when $\alpha(x)$ is known. For example in the case of HS1 pulses [3], with THK the thickness of the slice and β the cutoff factor of the pulse:

$$\alpha(x)=1/(2\beta)\log\left[(1+2x/THK)/(1-2x/THK)\right] \quad [5]$$

Methods

Numerical simulation of the Bloch equations was performed, using home-made programs written in Matlab (The Mathworks, Natick, MA, USA), for a pair of slice selective hyperbolic secant (HS1) pulses, with the time-bandwidth product R=60, pulse duration T_P=1 ms, echo time TE=10 ms (i.e. Δ=5 ms), and THK=1.5 mm. Time-step for the simulation was 5 µs. Simulation was performed for one million randomly diffusing spins, with diffusion coefficient D=5 µm^2/ms. Signal at the end of the sequence was then evaluated over 500 pixels spanning the slice thickness, by averaging the transverse magnetization over each pixel. Simulation was compared to the theoretical diffusion-weighted signal obtained when combining Eq.[4] and Eq.[5].

Results and discussion

The simulated signal attenuation along the direction of the slice selective gradient is shown in Fig. 2 (over 90% of slice thickness to exclude the transition bands). Simulation agrees very well with the theoretical attenuation, demonstrating the validity of Eq.[3] and subsequent equations. This confirms that the phase dispersion induced by the phase variation of the $\mathbf{B_1}$ field during the frequency sweep vanishes when considering the effect of this phase on diffusion-weighting, as predicted when calculating the phase gradient of the magnetization. In conclusion, only the phase dispersion induced by the slice selection gradient contributes to diffusion-weighting, which provides a convenient framework for b-values calculations. The only specific effect of frequency-swept pulses on diffusion-weighting arises from the (spatially dependent) instant t_Ω when the spins are flipped.

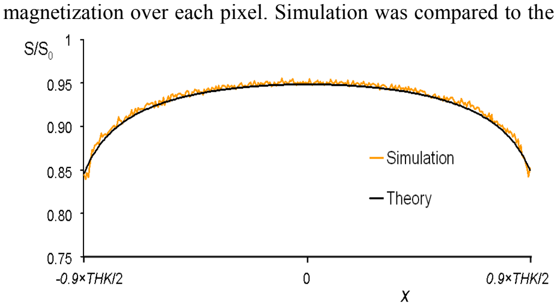

Fig.2: *Comparison between simulated and predicted signal loss after a pair of slice selective HS1 pulses, as a function of the position x along the thickness of the slice.*

[1] Silver *et al.*, J Magn Reson 1984; [2] Garwood and DelaBarre, J Magn Reson 2001; [3] Park *et al.*, Magn Reson Med 2006; [4] Sun and Bartha, J Magn Reson 2007; [5] Valette *et al.*, J Magn Reson 2007; [6] Park *et al.*, Magn Reson Med 2009.