Effect of Gradient Pulse Duration on Diffusion-Weighted Imaging Estimation of the Diffusional Kurtosis for the Kärger Model

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Introduction

Diffusion-weighted imaging (DWI) allows for the estimation of a number of diffusion metrics including the diffusion coefficient [1] and the diffusional kurtosis [2, 3]. In general, the accuracy of the estimates depends on the duration, δ, of the diffusion-sensitizing gradient pulses. For example, the apparent diffusion coefficient, D_{app}, and the apparent diffusional kurtosis, K_{app}, determined with a Stejskal-Tanner pulse sequence may, even in principle, only equal the true values, D and K, in the narrow pulse limit $\delta \to 0$ [2, 4]. However for D_{app}, prior studies of specific models with open restricted geometries have found only a modest dependence on δ [5]. Such results help to justify the use of DWI with larger pulse durations, which is often more practical especially for clinical applications. Here we utilize the Kärger model to investigate the effect of pulse duration on K_{app}. The Kärger model is a two-compartment diffusion model with exchange, which has been used to analyze DWI data for cell suspensions [6] and for brain [7].

Theory

The diffusional kurtosis for the Kärger model is given by [2]

$$K(T) = K_0 \frac{2\tau}{T}\left[1-\frac{\tau}{T}\left(1-e^{-T/\tau}\right)\right], \tag{1}$$

where T is the diffusion time, τ is the exchange time, and

$$K_0 = 3 f_a f_b \frac{(D_a - D_b)^2}{(f_a D_a + f_b D_b)^2}, \tag{2}$$

with D_a and D_b being the diffusion coefficients for the two compartments and f_a and f_b being the corresponding volume fractions. For a Stejskal-Tanner sequence, the apparent diffusional kurtosis can also be obtained analytically and shown to be

$$K_{app}(T,\delta) = \frac{2K_0}{15(X-Y/3)^2 Y^4}\left[15XY^4 - 9Y^5 - 40Y^3 + 60Y^2 - 120 + 120(Y+1)e^{-Y} + 120(Y-1)e^{-X} + 60(Y-1)^2 e^{-X+Y} + 60e^{-X-Y}\right], \tag{3}$$

where $X = T/\tau$ and $Y = \delta/\tau$. One may easily verify that $K_{app}(T, \delta) \to K(T)$ as $\delta \to 0$.

Methods

We define the percent error for the apparent diffusional kurtosis as: $100[K_{app}(T, \delta) - K(T)]/K(T)$. By using Eqs. (1) and (3), this error was calculated for $0 \le \delta/T \le 1$ and $0 \le T/\tau \le 10$. This includes the parameter range of physical interest for standard DWI of brain, since $0 \le \delta \le T$ is required for any Stejskal-Tanner sequence, $T \approx 20$ to 200 ms, and $\tau \approx 100$ ms [7].

Results

Figure 1 shows a contour plot of the percent error for K_{app}, which vanishes for either $\delta/T = 0$ or $T/\tau = 0$. The error also vanishes along a line corresponding approximately to $\delta/T = 0.85$; to the left of this line the error is positive, while to the right the error is negative. The maximum magnitude of the error is 6.2% for $\delta/T = 0.464$ and $T/\tau = 6.82$. As a specific example, Fig. 2 shows both $K(T)/K_0$ and $K_{app}(T, \delta)/K_0$ for $\delta/T = 0.6$ and $\tau = 100$ ms. In this case, a maximum error of 5.63% occurs for $T = 653$ ms.

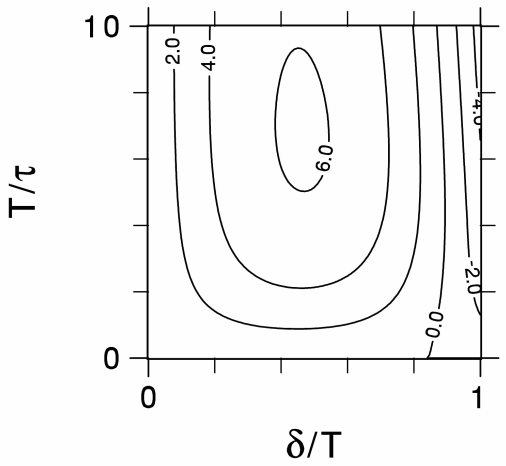

Figure 1. Contour plot of the percent error for the apparent diffusional kurtosis as calculated with the Kärger model and a Stejskal-Tanner sequence.

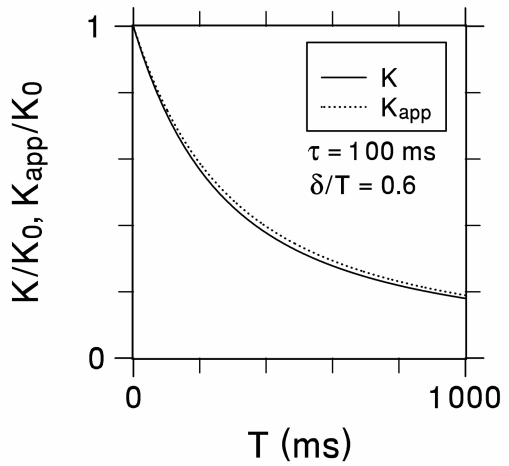

Figure 2. True (K) and apparent (K_{app}) diffusional kurtoses as functions of the diffusion time for δ/T = 0.6. The two curves nearly coincide demonstrating the insensitivity of K_{app} to the gradient pulse duration

Conclusion

For the Kärger model, the apparent diffusional kurtosis depends only weakly on the gradient pulse duration. In the parameter range appropriate for brain, K_{app} differs from K by at most a few percent. These results, while not definitive, provide support for the use of larger gradient pulse durations in the estimation of the diffusional kurtosis with DWI.

References: **1.** Bammer R. Eur J Radiol 2003;45:169. **2.** Jensen JH, et al. MRM 2005;53:1432. **3.** Lu H, et al. NMR Biomed 2006; 19:236. **4.** Tanner JE & Stejskal EO. J Chem Phys 1968; 49:1768. **5.** Zielinski LJ & Sen PN. J Magn Reson 2003;165:153. **6.** Roth Y, et al. Magn Reson Imaging 2008; 26:88. **7.** Lee JH & Springer CS Jr. MRM 2003;49:450.

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