Fast optimization method for general surface gradient coil design

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Introduction

Over the past 20 years many papers have discussed theoretical design methodologies for MRI gradient coils. She et al. [1] presented a method based on finite element method to design a gradient coil. Compare with the standard target field method, this is an attractive method because the design procedure can be employed to also consider the eddy current effects and an inhomogeneous analysis domain. However, the very large computational efforts induced by finite element analysis limit the practical application of this method.

In this paper, we present an efficient numerical iterative optimization method for designing linear gradient coil on a current-carrying surface. Using the scalar stream function as design variable, the value of the magnetic field inside a computational domain is calculated using the least square finite element method. The first-order sensitivity is calculated using the adjoint equation method. The detailed numerical optimization skills are discussed in order to obtain a fast and effective optimization procedure.

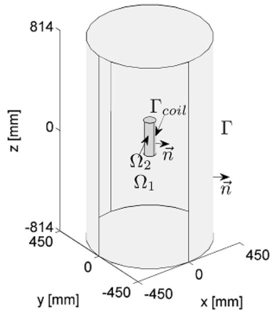

Figure 1. Current-carrying surface and computational domain.

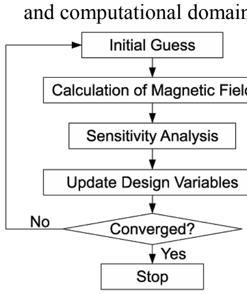

Figure 2. Flow chart of optimization.

Degree of Freedom	283500
Decomposition Time	319.1 s
Substitution Time	5.1 s

Table 1, The CPU time for matrix decomposition and substitution.

Theory

The optimization objective used in this paper is the least square type function $F=\int_{\Omega_{ROI}}(\frac{\partial B_z}{\partial x}-G_x)d\Omega_{ROI}$, where G_x is the linear x-gradient of specified magnetic field, Ω_{ROI} is the region of interest (ROI) and the magnetic field B_z is calculated using the Least-squares finite element method [2]. That is to find $\vec{B}_h=(B_x,B_y,B_z)^T\in V_h$ such that $a(\vec{B}_h,\vec{v}_h)=l(\vec{v}_h)$ for all $\vec{v}_h\in V_h$, where

$$a(\vec{B}_h,\vec{v}_h)=\sum_{i=1}^{2}\int_{\Omega_i}(\nabla\times\vec{B}_h\cdot\nabla\times\vec{v}_h+\nabla\cdot\vec{B}_h\cdot\nabla\cdot\vec{v}_h)d\Omega_i+\int_{\Gamma_{coil}}h^{-1}([\vec{B}_h\times\vec{n}]\cdot\vec{v}_h\times\vec{n}+[\vec{B}_h\cdot\vec{n}]\cdot\vec{v}_h\cdot\vec{n})ds+\int_{\Gamma}h^{-1}[\vec{B}_h\times\vec{n}]\cdot\vec{v}_h\times\vec{n}ds,$$

$$l(\vec{v}_h)=\int_{\Gamma_{coil}}\mu h^{-1}\vec{J}\cdot\vec{v}_h\times\vec{n}ds.$$

Here, V_h is the Lagrange finite element space of computational domain $\Omega=\Omega_1\cup\Omega_2$ (ROI is inside of Ω_2), h is the size of mesh, $\vec{n}$ is the unit normal vector on the boundary Γ or current-carrying surface Γ_{coil} (figure 1), $[u(x)]:=\lim_{s\to 0^+}u(x+s\vec{n})-u(x-s\vec{n})$ with $x\in\Gamma_{coil}$ denotes the jump of u across the Γ_{coil}, μ is the permeability and the surface current density $\vec{J}$ can be expressed as $\vec{J}=\nabla\times(\psi\vec{n})$ using the stream function ψ which is the design variables.

The design of gradient coil is an inverse problem. One needs to use the regularization technique to **avoid** the **oscillation** of the coil layout. Typically the inductance of the coil, or the magnetic energy term is combined to obtain a reasonable layout of the coil. In our example, we use the limited-memory BFGS method [3] with filter technique to implement the regularization effect. In the limited-memory BFGS, the first-order sensitivity of our objective can be obtained by the following formula $\partial \mathbf{L}/\partial\psi=\partial \mathbf{F}/\partial\psi+\alpha^T(\partial \mathbf{K}/\partial\psi)\mathbf{B}+\alpha^T(\partial \mathbf{J}/\partial\psi)$. Here $\mathbf{L}=\mathbf{F}(\mathbf{B}(\psi),\psi)+\alpha^T(\mathbf{J}-\mathbf{KB})$ is the corresponding discretized Lagrangian model of the original optimization problem, $\mathbf{F}$ is the discretized expression of objective function in equation (1), $\mathbf{B}$ is the unknown vector of magnetic field, α is the Lagrange multiplier, $\mathbf{J}$ is the discretized vector of surface current density and $\mathbf{K}$ is the discretized global stiffness matrix using the LSFEM. The Lagrange multiplier α can be obtained by solving the following adjoint equation $\mathbf{K}^T\alpha=\partial\mathbf{F}/\partial\mathbf{B}$.

An iterative optimization of gradient coil design includes several key steps which are run sequentially (figure 2). The main computational cost for each iteration is the steps of the finite element analysis and sensitivity analysis. When the LSFEM is used to discretize a design domain, the discretized stiffness matrix $\mathbf{K}$ is kept unchanged and merely needs to be assembled once for all iterative loops. Therefore only the vector on the right hand side of the discretized equilibrium equation needs to be updated in each iteration. Based on this condition, one can perform the decomposition of the matrix $\mathbf{K}$ merely once and save the decomposed matrices at the beginning of the optimization. Then only back substitution is performed to obtain the solution of LSFEM and sensitivity. Because the time used for substitution step is much shorter than that used for the decomposition step (table 1), this strategy can be employed to speed up the whole optimization procedure.

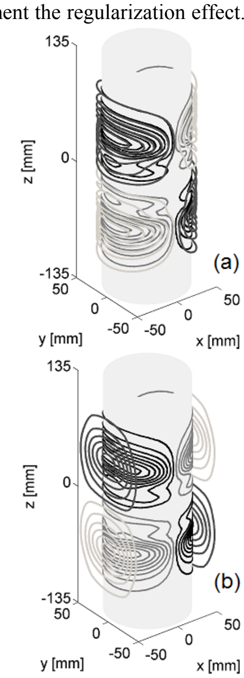

Figure 4 Layouts of the Gx gradient coils using (a) two cylinder surface coils and (b) one cylinder and one planar surface coil.

Numerical results

Figure 3 shows an example of a Gx gradient coil on a cylindrical surface Γ_{coil} with radius=45mm and height=270mm. The ROI is a cylinder with radius=30mm and height=42mm. The whole domain is discretized into a hexahedral mesh with 89610 points. The Gx gradient coil layout (contour lines of the stream function) on the Γ_{coil} is shown in figure 3a. The gradient strength at the center of ROI is 10mT/m. Figure 4 shows layouts of two multi-layer Gx gradient coils for target gradient strength =10mT/m.

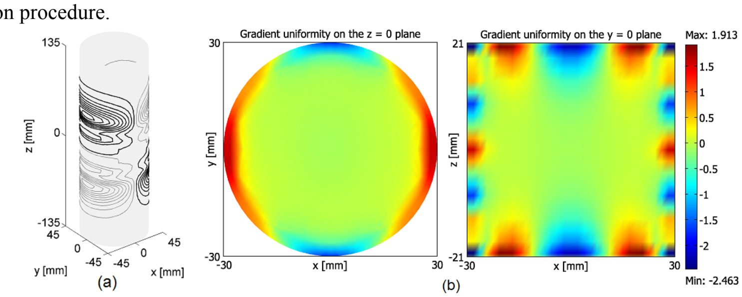

Figure 3, (a) Layout of G_x gradient coil with current = 4.5824 A. (b) Percentage deviation of $\partial B_z/\partial x$ from 10mT/m on z=0 (left) and y=0 (right) plane in the ROI.

Discussion

This abstract presents a **fast optimization procedure** to design a **surface gradient coil** for MRI. Based on LSFEM, numerical examples demonstrate that this method can be used to design a gradient coil on any surface.

Reference

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