The intrinsic magnetic field symmetries of the spiral birdcage coil

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INTRODUCTION Transmit array spatial encoding (TRASE) is a novel MRI technique that utilizes B_1-field phase gradients as the means for traversing k-space [1-3]. This method allows MRI to be performed without the usual application of B_0-field amplitude gradients, thereby possibly eliminating the equipment, complexity, power consumption and acoustic noise associated with traditional *static-field* gradient coils. As a result, TRASE might provide a comfortable (i.e. silent) imaging experience in a lower-cost, simplified scanner.

The primary requirement of TRASE is an RF phase-gradient coil, which ideally generates a transverse RF field of constant magnitude and linearly varying phase over the sample volume. Previous work [1-3] did not focus on the design and optimization of RF phase-gradient coils. Rather, basic coil structures with roughly the correct field and phase symmetries were employed. Most notably, a spiral birdcage coil (Fig. 1) was used to generate a transverse RF field with phase varying along B_0. Here we explore the intrinsic properties of such a coil in order to guide and motivate the future design of phase-gradient coils.


Fig. 1: An 8-leg spiral birdcage coil with total twist of 2π.

APPROACH To begin, one must ask the question: does the ideal, transverse, phase-gradient field $\mathbf{B_1}=B_1(\cos(z)\mathbf{i} + \sin(z)\mathbf{j})$ even satisfy Maxwell's equations? Unlike its counterpart $\mathbf{B_1}=B_1\mathbf{i}$ associated with conventional RF production by standard birdcage coils, the answer is no – at the very least, it turns out, a z-component must accompany the desired transverse field here.

To more thoroughly analyze the spiral birdcage coil, we use a Fourier transform method that was originally developed to design self-shielded gradient coils [4]. We previously used this technique to design active and passive shielding for contiguous, wire-wound, RF coils [5,6] for very low field MRI; and we show here that the same approach can in fact be used to model the multi-connected, resonant structures of birdcage coils, as well.

The method comprises the following steps: (i) the coil structure is modeled as a current distribution on a cylindrical surface; (ii) the Fourier transform of the surface current is determined; (iii) the Fourier components of the surface current are substituted into Eq. (4) of Ref. [5], which provides a compact, analytic form of the magnetic field vector generated at any point in free space. The temporal dependence of the currents can also be included to explore linear versus quadrature coils. A very useful feature of this analysis is that for any azimuthally-symmetric, birdcage-like structure not operating in an end-ring mode, the net magnetic field can be completely specified by knowledge of the axial currents only – and these, as we show, are quite easy to write down.

RESULTS The archetypal surface current distribution for the standard birdcage coil is the well-known, infinitely-long, sine-phi distribution, which can be written as $\mathbf{F} = \mathbf{F_z} = F_1 \sin(\phi)\,\mathbf{k}$. Its Fourier components are $F^m = im\pi F_1$ (with m=±1 only) and its magnetic field (by Eq.(4) of Ref. [5]) is $\mathbf{B_1} = \mu_o F_1/2\,\mathbf{i}$, which has uniform magnitude and phase and is independent of z. The endeavor of standard birdcage design is to approximate this surface current and field as well as possible using a finite-length coil with discrete current paths.

The archetypal, infinitely-long, surface current distribution for the spiral birdcage coil has an axial component that can be written as $\mathbf{F_z} = F_1 \sin(\phi + hz)\cos(\alpha)\,\mathbf{k}$, where h is the twist-per-unit-length of the coil and α is the pitch angle of the spiral. Here the Fourier components of the axial surface current are $F^m = -im\pi F_1\cos(\alpha)$ (with m=±1 only) and the magnetic field inside the coil evaluates to

$$\begin{pmatrix} B_\rho \\ B_\phi \\ B_z \end{pmatrix} = \mu_o F_1 a^2 \cos(\alpha) K_1'(ha) \begin{pmatrix} -h^2 \cos(hz+\phi)\; I_1'(h\rho) \\ h/\rho \sin(hz+\phi)\; I_1(h\rho) \\ h^2 \sin(hz+\phi)\; I_1(h\rho) \end{pmatrix}$$

where I_1 and K_1 are modified Bessel functions (with prime denoting derivative) and a is the coil radius. Here B_z is non-zero and its magnitude depends on the radial position ρ. More importantly, the transverse field components are not uniform for a given z, but also vary as a function of ρ. Near the central axis, as $\rho\to 0$, the variation in these field components scales as ρ^2. The increase in magnitude of the transverse field (and hence also sensitivity) away from $\rho=0$ is consistent with the experimental observation that spiral birdcage coils exhibit less central image brightening [7].

For a more sophisticated analysis, the axial component of the surface current of an actual spiral birdcage coil can be modeled as

$$F_z = \frac{1}{a}\cos(\alpha)\;\Pi(-l,l)\sum_{j=1}^{N} I_j(t)\;\delta(\phi - \Phi_j(z))$$

where Π is the boxcar function truncating the coil at half-lengths $\pm l$, N is the number of spiral rungs, I_j is time dependent current in each rung and the delta function sets the azimuthal location of each rung at angle $\Phi_j(z)$. The Fourier transform of Fz and a complete analytic formulation of the spiral birdcage field follow quickly from this. As above, we find that even near the coil isocentre there is a strong radial dependence in the transverse field.

CONCLUSION The spiral birdcage coil and its archetypal surface current distribution (the infinitely-long, twisted, sine-phi) have been analyzed using the Fourier transform method laid out in Refs. [4,5]. The results show that twisting leads to a strong, intrinsic radial dependence in all magnetic field components. This knowledge will be used to guide the optimization of the next generation of phase gradient coils used for TRASE. It may also provide the basis of a more direct correction scheme for the problem of central brightening that occurs in high field imaging [7].

REFERENCES [1] JC Sharp and SB King, Magn Reson Med (in press). [2] SB King *et al*., Proc ISMRM, 2628 (2006) & 680 (2007). [3] JC Sharp *et al*., Proc ISMRM, 829 & 1083 (2008). [4] R Turner and RM Bowley, J Phys E **19**, 876 (1986). [5] CP Bidinosti *et al*., J Magn Reson **177**, 31 (2005). [6] CP Bidinosti and ME Hayden, Applied Phys Lett **93**, 174102 (2008). [7] DC Alsop *et al*., Magn Reson Med **40**, 49 (1998).